Parrilo asked: Are all convex forms sos?

Blekherman: No

For $d \geq 4$ there are many more convex forms than sos as $n \to \infty$

No explicit examples are known!

Minimal suspects: $(n, d) = (3,6)$ and $(n, d) = (4,4)$
**Theorem:** Convex forms in \((n, d) = (4, 4)\) are sos.

Same holds for \((n, d) = (3, 6)\) if a conjecture of Blekherman (2012) is true.

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**Idea of the proof:**
Generalized Cauchy-Schwarz

\[ p(x) = x^T Q x \quad Q \text{ symmetric} \]

If \( p \) is convex, then

\[ x^T Q y \leq \sqrt{p(x)p(y)} \quad \forall x, y \]

\[ p(x) = \langle T, x^{\otimes 4} \rangle \quad T \text{ symmetric} \]

If \( p \) is convex, then

\[ \langle T, x^{\otimes 2} \otimes y^{\otimes 2} \rangle \leq \sqrt{p(x)p(y)} \quad \forall x, y \]

Theorem: If \( p \) is convex form of degree 4, then

\[ \frac{1}{12} y^T \nabla^2 p(x) y \leq \sqrt{p(x)p(y)} \quad \forall x, y \]
- SOS is an important tool for MURI

- We need a better understanding of the gap between SOS and nonnegative polynomials, especially for *structured* problems

- Today: Interplay between *convexity* and SOS

Thanks!