A Brief Introduction to Formal Methods

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Formal Methods

Formal Methods: mathematically/logically - based techniques for the specification, development and verification of SW/HW systems.

Key ideas:

- *Analysis/synthesis* of *models* of systems
- *Precise* specification of intended behavior

Main Focus: Reactive System

- ***Reactive System***: a system that reacts to external events
- Formal Model – *Transducers*

$T=(S,s_0, C,D,f,g)$

- S: finite state set, s_0 – initial state
- C: input symbols, D: output symbols
- f: SxC->S – transition function
- g: S->D - output function

Trace: $(s_0,c_0,d_0),(s_1,c_1,d_1), \ldots$

- $s_{i+1}=f(s_i,c_i)$, $d_i=g(s_i)$

Nondeterminism

- ***Nondeterminism***: modeling *uncertainty*
- Formal Model – Nondeterministic *Transducers*

T=(S,s0, C,D,f,g)

- S: state set, s0 – initial state
- C: input symbols, D: output symbols
- f: $S \times C \rightarrow 2^S$ – *nondeterministic* transition fun.
- g: S->D - output function

Trace: $s_{i+1} \varepsilon f(s_i, c_i)$

- Note: *probabilistic* treatment to come

Specification

Basic Approach: specify *what,* rather than *how.*

- *In particular:* specify properties of traces

Examples: *Linear Temporal Logic* (LTL)

- always not (CS1 and CS2): mutual exclusion -- safety
- always (Request implies eventually Grant) – liveness

Psalm 34:14: "Depart from evil and do good"

Analysis and Synthesis

- ***Analysis Problem***: Given a *finite* transducer T and an LTL formula φ, do *all* traces of T satisfy φ? That is, is T a *model* of φ? (*Model Checking*)
- ***Synthesis Problem***: Given an LTL formula φ, *is there* a finite transducer T that is a model of φ? If so, construct one.

Analysis and Synthesis - Results

- ***Analysis Problem***: Given a *finite* transducer T and an LTL formula φ, do <u>*all*</u> traces of T satisfy φ? That is, is T a model of φ? (*Model Checking*)
  - Problem is PSPACE-complete.
- ***Synthesis Problem***: Given an LTL formula φ, is there a finite transducer T that is a model of φ? If so, construct one.
  - Problem is 2EXPTIME-complete

Key Algorithmic Ideas - Automata

- Models(φ) is a subset of $(2^{Prop})^{\omega}$
  - A formal language of infinite words
  - Can also be described by an automaton in infinite words
  - Algorithmic translation from logic to automata
- Mature algorithmic theory for analysis
  - Academic and industrial tools

Verification and Planning

- Analysis Problem: Given a finite transducer T and an LTL formula φ, do all traces of T satisfy φ?
- Answer:
  - T is a model of φ – MC says "Yes"
  - T is not a model of φ; MC says "No" and provides a counterexample trace
- ***Bonus***: An algorithm to find system trace that satisfies a temporal property – *planning*!
  - Applications to robotics.

Key Algorithmic Ideas - Games

- **Synthesis**: Can be viewed as an infinite-round *game* between system and environment
  - Environments chooses input symbols.
  - System chooses output symbols.
  - System wants to satisfy φ.
  - System wins if it has a *winning strategy*.
  - Strategy can be written as a transducer.
- Elegant algorithmic theory for synthesis
  - Academic tools

Beyond Finite Systems

- ***Model Checking***: Is L(T) contained in L(φ)?
- ***Language Containment***: L_1 contained in L_2?
  - Decidable when both L_1 and L_2 are regular.
  - Undecidable if L_2 is context free.
- Beyond finite systems:
  - Model checking of recursive systems
  - Wrt "weak recursive" LTL (i.e., match calls and returns)
- Academic and industrial tools

Fundamental Challenge: Scalability

- Theoretical/practical computational complexity
  - Theoretical: *worst case* -- too pessimistic!
  - Practical: algorithmic behavior on real-world problem instances
- ***Good news***: We can solve industrial problems of nontrivial size
- ***Bad news***: It is hard to overcome exponential scaling – "climbing Mount Exponential"
  - "State-Explosion Problem"

Revolutionary Progress – SAT Solving

- Boolean Satisfiability (SAT): Given a Boolean expression, using "and" (&) "or", (|) and "not" (!), is there a satisfying solution (an assignment of 0's and 1's to the variables that makes the expression equal 1)?
- Example:

(¬x1 |x2 |x3) & (!x2 | !x3 |x4) & (x1 | x3 |x4)

- Solution: x1 = 0, x2 = 0, x3 = 1, x4 = 1

Modern SAT Solving

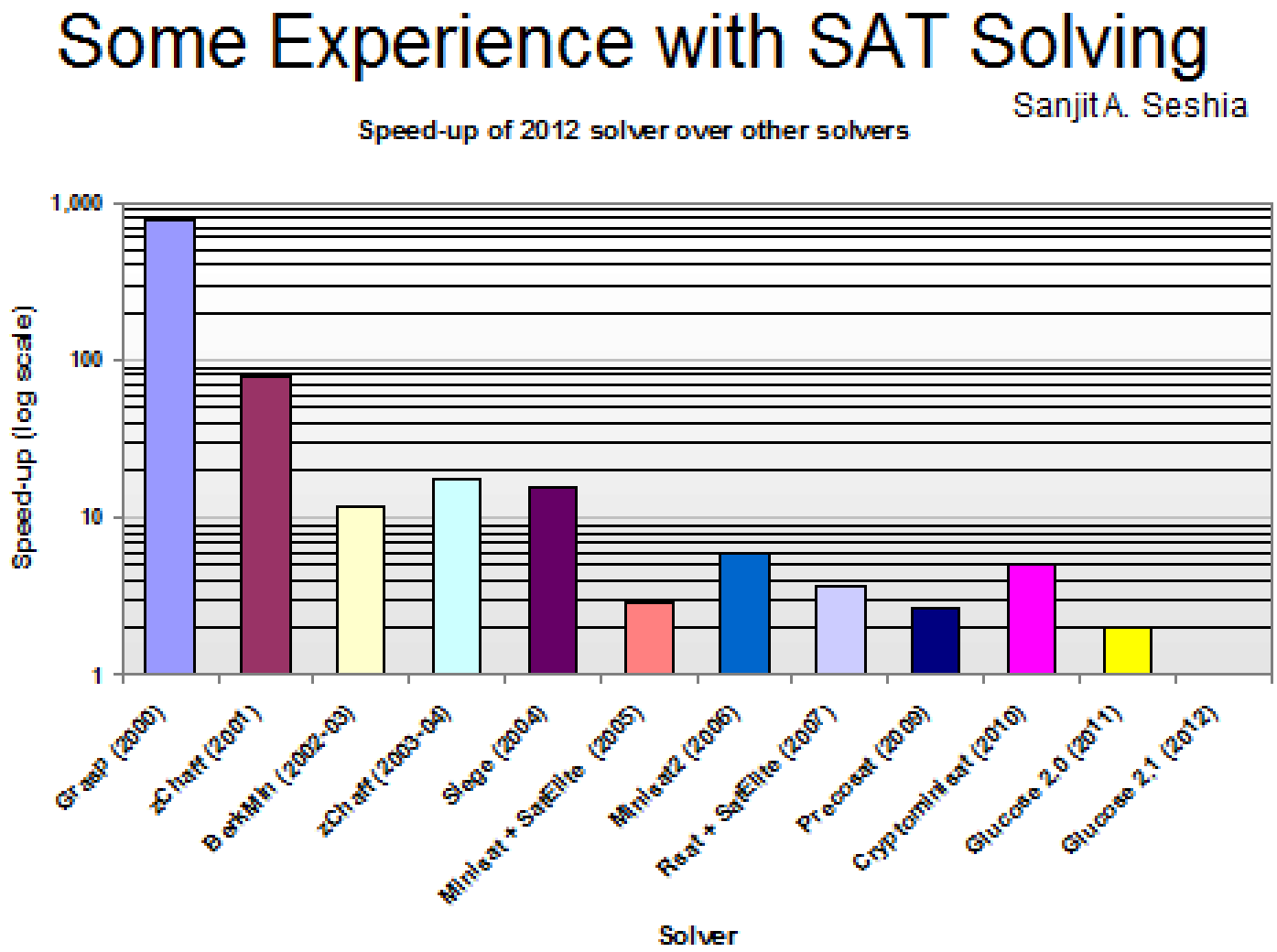
CDCL = conflict-driven clause learning

- Backjumping
- Smart unit-clause preference
- Conflict-driven clause learning
- Smart choice heuristic
- Restarts

Key Tools: GRASP, 1996; Chaff, 2001

Current capacity: *millions* of variables

“Moore’s Law for SAT Solving”



Summary

Formal Methods:

- Motivated by real-world problems
- Rich algorithmic theory
- Academic and industrial tools
- Impressive progress wrt real-world complexity
- Scalability challenge
- ***Current frontier***: *quantitative* analysis