The closed-loop transfer function is

$$T(s) = \frac{K}{s^2(s + p) + K}.$$ 

Therefore, the characteristic equation is

$$s^3 + ps^2 + K = 0.$$
The Routh array is

\[
\begin{array}{ccc}
 s^4 & 1 & 0 \\
 s^3 & p & K \\
 s^2 & -K/p & \\
 s^1 & K & \\
\end{array}
\]

We see that the system is not stable for any value of \( p \) and \( K \).
(a) The closed-loop characteristic equation is

\[ 1 + \frac{K}{(0.5s + 1)(s + 1)(\frac{1}{2}s + 1)} = 0, \]

or

\[ s^3 + 7s^2 + 14s + 8(1 + K) = 0. \]

The Routh array is
\[
\begin{align*}
\text{s}^3 & \quad 1 & 14 \\
\text{s}^2 & \quad 7 & 8(1 + K) \\
\text{s} & \quad b & b \\
\text{s}^0 & \quad 8(1 + K) & 8(1 + K) \\
\end{align*}
\]
where

\[
b = \frac{7(14) - 8(1 + K)}{7} .
\]

For stability, we require \( b > 0 \) and \( 8(1 + K) > 0 \). Therefore, the range of \( K \) for stability is

\[-1 < K < 11.25 .\]

(b) Let \( K = 11.25/3 = 3.75 \). Then, the closed-loop transfer function is

\[
T(s) = \frac{3.37}{\text{s}^2 + 7\text{s}^2 + 14\text{s} + 38} .
\]
The settling time to a step input is \( T_s \approx 6 \) seconds.

(c) We want \( T_s = 4 \) sec., so

\[
T_s = 4 = \frac{4}{\zeta\omega_n} \quad \text{implies} \quad \zeta\omega_n = 1 .
\]

Our desired characteristic polynomial is

\[(s + b)(s^2 + 2\zeta\omega_n s + \omega_n^2) = s^3 + (2 + b)s^2 + (\omega_n^2 + 2b)s + b\omega_n^2\]

where we have used the fact that \( \zeta\omega_n = 1 \) and \( \omega_n \) and \( b \) are to be determined. Our actual characteristic polynomial is

\[s^3 + 7s^2 + 14s + 8(1 + K) = 0 .\]
Comparing the coefficients of the actual and desired characteristic polynomials, we find the following relationships

\[2 + b = 7 \quad \omega_n^2 + 2b = 14 \quad b\omega_n^2 = 8(1 + K) .\]

Solving these three equations yields

\[b = 5, \quad \omega_n = 2 \quad \text{and} \quad K = 1.5 .\]
The actual settling time is \( T_s = 4.17 \) sec. This is not exactly our
desired $T_e$ since we have the contribution of the additional pole at $s = -5$. The closed-loop poles are

$$s_1 = -5 \quad \text{and} \quad s_{2,3} = -1 \pm 1.73j.$$
The Routh array is:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>31</th>
<th>0.5K</th>
</tr>
</thead>
<tbody>
<tr>
<td>s^3</td>
<td>1</td>
<td>K - 41</td>
<td></td>
</tr>
<tr>
<td>s^2</td>
<td>320K</td>
<td>0.5K</td>
<td></td>
</tr>
<tr>
<td>s^1</td>
<td>b</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>s^0</td>
<td>0.5K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where

\[ b = \frac{(320 - K)(K - 41) - 40.5K}{320 - K} \]

Therefore, using the condition that \( b > 0 \), we obtain the stability range for \( K \):

\[ 48.178 < K < 272.32 \]
The closed-loop transfer function is

$$T(s) = \frac{K}{s^3 + 10s^2 + 20s + K}.$$ 

The range of $K$ for stability is $0 < K < 200$. If we let $K = \frac{K_m}{N}$ where $K_m = 200$, then $N = 6.25$ results in a step response with $P.D.E. = 15.7\%$ and $T_e = 1.96$ seconds.
The closed-loop system poles for the slow/fast pilots are shown in Figure MP6.5. The maximum allowable time delay is 0.1776 seconds. At the maximum allowable time delay, the system has roots on the $j\omega$-axis at $\pm 2.1$. The slow pilot destabilizes the aircraft.

close-loop

\[
\begin{align*}
\text{system poles} & : \\
\text{fast pilot} & = -0.3293 - 1.209i \quad -0.3293 + 1.209i \\
& = -0.6586 \\
& = -0.2102 - 2.414i \\
& = -0.2102 + 2.414i \\
& = -0.3679
\end{align*}
\]

\[
\begin{align*}
\text{slow pilot} & = -0.3098 - 1.102i \quad -0.3098 + 1.102i \\
& = -0.6196 - 1.206i \\
& = 0.0116 + 2.074i \\
& = 0.0116 - 2.074i \\
& = -0.3754
\end{align*}
\]

\[
\begin{align*}
\text{maximum pilot delay} & = 0.4776 \text{ sec} \\
\text{slow pilot, $\tau = 0.4776$:} & = -0.3098 - 1.102i \quad -0.3098 + 1.102i \\
& = -0.6196 - 1.206i \\
& = 0.0116 + 2.074i \\
& = 0.0116 - 2.074i \\
& = -0.3754
\end{align*}
\]

**Figure MP6.5**

Closed-loop system poles for an aircraft with a pilot in the loop.