The closed-loop transfer function is
\[
T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} = \frac{100K_1(s + 5)}{s^2 + 105s + (500 + 100K_1K_2)}.
\]

The steady-state tracking error is
\[
E(s) = Y(s) - R(s) = \frac{G_c(s)G(s)(1 - H(s)) - 1}{1 + G_c(s)G(s)H(s)} R(s)
= \frac{-s^2 - (105 - 100K_1)s - 500 + 100K_1(5 - K_2)}{s^2 + 105s + 500 + 100K_1K_2} \cdot \frac{1}{s}
\]
and
\[
\lim_{s \to 0} sE(s) = \frac{-5 + K_1(5 - K_2)}{5 + K_1K_2}.
\]

(b) The transfer function from the noise disturbance \(N(s)\) to the output \(Y(s)\) is
\[
Y(s) = \frac{-G_c(s)G(s)H(s)}{1 + G_c(s)G(s)H(s)} N(s) = \frac{-100K_1K_2}{s^2 + 105s + (500 + 100K_1K_2)} N(s).
\]
The steady-state error to a unit step \(N(s) = 1/s\) is
\[
\lim_{s \to 0} sY(s) = \lim_{s \to 0} s \cdot \frac{-100K_1K_2}{s^2 + 105s + (500 + 100K_1K_2)} \cdot \frac{1}{s} = \frac{-K_1K_2}{5 + K_1K_2}.
\]

(c) The design trade-off would be to make \(K_1K_2\) as large as possible to improve tracking performance while keeping \(K_1K_2\) as small as possible to reject the noise.
The closed-loop transfer function is
\[ T(s) = \frac{G_c G(s)}{1 + G_c G(s)} = \frac{K}{(10s+1)(2s+1) + K} = \frac{K}{20s^2 + 12s + 1 + K}. \]

(b) The sensitivity \( S_T \) is
\[ S_T = \frac{\frac{\partial T}{\partial K}}{\frac{\partial T}{\partial K} \cdot \frac{K}{T}} = \frac{1}{1 + G_c G(s)} = \frac{1}{1 + \frac{1}{10s+1 + 2s + 1}}. \]

(c) Define \( E(s) = R(s) - Y(s) \). Then
\[ E(s) = \frac{R(s)}{1 + G_c G(s)} = \left[ \frac{25s^2 + 12s + 1}{20s^2 + 12s + 1 + 1} \right] R(s). \]

With
\[ R(s) = \frac{4}{s}, \] we find that the steady-state error
\[ e_{ss} = \lim_{s \to 0} s E(s) = \frac{4}{1 + K}. \]
D34.3 (a) The closed-loop transfer function is

\[ T(s) = \frac{\omega(s)}{\mu(s)} = \frac{K}{s^2 + 5s + KK_1}. \]
\[ E(s) = \left(1 - \frac{1}{s} T(s)\right) \omega(s) = \frac{s^2 + 5s + K(K_1 - 1)}{s^2 + 5s + K K_1} \]

So, if
\[ 0.99 < K_1 < 1.01 \]
then
\[ \omega_{\text{sat}} < 0.01 \]

(b) The transfer function from \( D(s) \) to \( \omega(s) \) is
\[ \omega(s) = \frac{-s}{s^2 + 5s + K K_1} D(s) \]

So, with \( E(s) = -\omega(s) \) and \( D(s) = 2/s \), we have
\[ \lim_{s \to 0} sE(s) = \frac{2}{K K_1} \]

Therefore, we select \( K K_1 > 20 \) to obtain \( \omega_{\text{sat}} < 0.1 \).
The step responses for the proportional and PI controller are shown in Figure MP4.7. The steady-state tracking error for the proportional controller is

\[ \varepsilon_{ss} = 0.33 \]

Increasing the complexity of the controller from a proportional controller to a proportional plus integral (PI) controller allows the closed-loop system to track the unit step response with zero steady-state error. The cost is controller complexity, which translates into higher costs (\$).
\[ K_o = \lim_{s \to 0} sC(s) = \lim_{s \to 0} s \left[ \frac{75(s + 1)}{s(s + 5)(s + 20)} \right] = \frac{75}{100} = 0.75 , \]

and

\[ c_{so} = \frac{\frac{100}{K_o}}{0.75} = 1.33 . \]