1. (40pts)
   a. (30) Derive the equations of motion for climbing flight at constant power setting and constant velocity in a vertical plane over a flat earth. Start from the definitions of velocity, acceleration, Newton’s second law, and weight rate, and derive the equations of motion in the wind axes system. Derive the equations for this specific scenario; do not derive the equations in general and then simplify them to this scenario.
   b. (10) How many mathematical degrees of freedom do the equations of motion in part (a) have?

2. (20pts) Consider an Ideal Subsonic Airplane (ISA) in a quasi-steady climb at constant power setting. Derive the equation for the time to climb from $h_0$ to $h_f$ in terms of the unknown velocity profile $V(h)$. Recall that the weight is assumed to be constant on the RHS of the EOM.

3. (25pts) The horizontal distance traveled during quasi-steady climb at constant power setting of an ISA from $h_0$ to $h_f$ is given by

$$x_f - x_0 = \int_{h_0}^{h_f} \frac{dh}{\gamma}$$

where

$$\gamma = \frac{1}{W} \left[ T_*(P) \left( \frac{\rho}{\rho_*} \right)^a - \frac{1}{2} C_D_0 \rho S_W V^2 - \frac{2KW^2}{\rho S_W V^2} \right].$$

   a. (10) Derive the velocity profile that should be flown to minimize the distance?
   b. (15) If the airplane is flown at constant equivalent airspeed ($V_e = \sqrt{\sigma V^2}$) in an exponential atmosphere carry out the derivation of the distance to the point that an integral can be looked up in a table, that is,

$$x_f - x_0 = \int f(x) dx.$$
4. (20pts) Consider an airplane in nonsteady gliding flight \((T = 0)\). Assume that \(\gamma\) and \(\dot{\gamma}\) are small but that \(\dot{V}\) is not. Derive the expression for the time to descend from one energy level to a lower energy level using the velocity profile \(V(E_s)\).

5. (35pts) An airplane is in a steady glide at \(h=30,000\) ft \((\rho=.0009 \text{ slug/ft}^3, a=990\text{ft/s})\), \(M=0.6\) and \(W=11,000\) lbs. Its planform area is 250 ft\(^2\), and its mean aerodynamic chord is 7.0 ft. At this flight condition, the aerodynamic characteristics are given by

\[
C_D = 0.02 + 0.07C_L^2 \\
C_L = 0.09 + 6.0\alpha + 0.5\delta_E \\
C_m^A = 1 + 6.0(\bar{X}_{cg} - .6)\alpha - 1.0\delta_E \\
C_h = .006 - .08\alpha - .5\delta_E - .2\delta_T
\]

a. (10) For what cg positions is the airplane statically stable? If \(\bar{X}_{cg} = .4\), what is the value of the static margin? Where is the neutral point located? Where is the aerodynamic center located?

c. (25) If \(\bar{X}_{cg} = .4\), what is the elevator angle at this flight condition? What trim tab deflection is needed to give zero stick force at this flight condition? Does the sign of \(\delta_T\) make sense?

6. (40pts) Define briefly the following items using words, formulas, figures, etc.

\[
\begin{align*}
a. \text{stability axes} & \quad e. \text{trimmed drag polar} \\
b. \text{zero – lift drag} & \quad f. \text{corner speed} \\
c. \text{lift – off speed} & \quad g. \text{C}_L^{\text{max}} \\
d. \text{taper ratio} & \quad h. \text{planform area}
\end{align*}
\]

7. (20pts) An airplane is in steady level flight when the pilot inputs an elevator deflection. The perturbed roll angle is governed by the equation

\[\ddot{\phi} + 0.44\dot{\phi} = 6.8\delta_A.\]

At \(t = 0\), the roll angle perturbation, the roll angle rate perturbation, and the aileron angle perturbation satisfy the relations

\[\phi(0) = 0, \quad \dot{\phi}(0) = 0, \quad \delta_A(0) = 0.\]

What is the roll response to an aileron impulse? Is the response stable? What is the steady state response?