Fracture and fatigue

- Fast fracture: $K > K_c$, catastrophic failure.

- High-cycle fatigue: crack initiation by cumulative damage.

- Fatigue crack propagation: $K < K_c$, subcritical growth.

- Static fatigue: environment assisted subcritical crack growth.
Concepts of Fracture Mechanics

- K-field characterizes the stresses near crack tip.
- Same K-field implies same crack tip process.

\[ K = \sigma \sqrt{\pi a} \]

In general:

\[ K = Q(a,\ldots) \sigma \sqrt{\pi a} \]
Cyclic K-history

\[ \Delta K = K_{\text{max}} - K_{\text{min}} \]

\[ R = \frac{K_{\text{min}}}{K_{\text{max}}} \]

- Over one cycle, neglect the crack extension:

\[ \Delta K = \Delta \sigma \sqrt{\pi a} Q(a) \]

- \( \Delta K \) varies from cycle to cycle.
Fatigue crack propagation

- Measure $a$ vs $N$ at different stress levels.
- Calculate crack growth rate and $\Delta K$. 
Fatigue crack growth rate

- Data from different stress levels collides into one single curve.
- Three distinct regions.
Crack growth regions

- (I) below threshold: no growth;
- (II) steady-state growth: power law;
- (III) Fast fracture
Paris Law

• Most of the life time of pre-cracked material is spent in the steady-state region.

\[
\frac{da}{dN} = C(\Delta K)^m
\]

\(C, m\) are material constants determined from experiments.
Typically, \(m = 4\).
Fatigue life prediction

- Initial crack length: $a_0$
- Critical crack length: $a_c$
- Number of cycles to fail (lifetime):

$$N_f = \int_{a_0}^{a_c} \frac{da}{C(\Delta K)^m}$$
Example

\[
K = \sigma \sqrt{\pi a} \quad \rightarrow \quad a_c = \frac{1}{\pi} \left( \frac{K_c}{\sigma_{\text{max}}} \right)^2
\]

Assume \( W >> a_c >> a_0 \)

\[ \Delta K = \Delta \sigma \sqrt{\pi a} \]

\[
N_f = \int_{a_0}^{a_c} \frac{da}{C(\Delta K)^m} = \frac{2}{C(\Delta \sigma)^m \pi^{m/2} (m-2)} \left[ \frac{1}{a_0^{m/2-1}} - \frac{1}{a_c^{m/2-1}} \right]
\]

- See a worked example in Chapter 16 of A&J1.
Cycle-by-cycle growth

$$\Delta K = \Delta \sigma \sqrt{\pi} a Q(a)$$

- For cycle $i = 0, 1, 2, \ldots$

$$\left(\Delta a\right)_i = C \left(\Delta K_i\right)^m = C \left[\Delta \sigma \sqrt{\pi} a_i Q(a_i)\right]^m$$

$$a_{i+1} = a_i + \Delta a_i$$

Fail if $a_{i+1} \geq a_c$. 
Fatigue life

- Three-stage fatigue life: crack nucleation, crack propagation, and final failure.

The nucleation stage is shortened drastically or completely eliminated with pre-existing flaws.
Fatigue fracture beachmarks

**Figure 8.19** Fracture surface of a rotating steel shaft that experienced fatigue failure. Beachmark ridges are visible in the photograph. (Reproduced with permission from D. J. Wulpi, *Understanding How Components Fail*, American Society for Metals, Materials Park, OH, 1985.)
Fatigue striation

Fatigue induced fast fracture

**Figure 8.21** Fatigue failure surface. A crack formed at the top edge. The smooth region also near the top corresponds to the area over which the crack propagated slowly. Rapid failure occurred over the area having a dull and fibrous texture (the largest area). Approximately 0.5×.