ASE324: Aerospace Materials Laboratory

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Fall 2003
Bending of a glass rod

\[ d = 10 \text{ mm} \]
\[ E = 100 \text{ GPa} \]
\[ K_c = 1 \text{ MPa-m}^{1/2} \]
Energy approach in fracture mechanics

- Elastic deformation: strain energy
- Plastic deformation: energy dissipation
- Crack opening: surface energy

- Energy release rate versus fracture resistance
Strain energy

- Static load, no dynamic or inertia effects.
- Work done by the load equals the strain energy stored in the bar (energy conservation).

\[ U = W = \int_{0}^{\delta} P d\delta \]
Linear elastic strain energy

\[ U = W = \frac{1}{2} P \delta = \frac{P^2 L}{2EA} = \frac{EA \delta^2}{2L} \]

Strain energy density:

\[ u = \frac{U}{AL} = \frac{\sigma^2}{2E} = \frac{E\varepsilon^2}{2} \]

- The bar acts as an elastic spring, storing and releasing energy as loading and unloading.
Energy dissipation

- Plastic deformation dissipates energy, i.e., the energy that is not recovered due to permanent deformation.
Surface energy

- Surface energy: excess energy at a surface or an interface relative to the body.

Stretching a liquid membrane: liquid molecules move to surface to increase the surface area.

Work done by load $F = \text{increase of surface energy}$

$$F \Delta L = 2\gamma W \Delta L$$

$$\gamma = \frac{F}{2W}$$

No friction

What if it is a solid (e.g., rubber) membrane?
Capillary effect

\[ \gamma_1 = \text{surface energy density of the interface between the tube and air} \]

\[ \gamma_2 = \text{surface energy density of the interface between the tube and the liquid} \]

Change of \( h \) leads to change of the surface energies and the potential energy due to gravity.

\[ G = G_0 + (\gamma_2 - \gamma_1)2\pi ah + \rho g \pi a^2 h \cdot \frac{h}{2} \]

At equilibrium, the total energy is the minimum. Thus

\[ \frac{\partial G}{\partial h} = 0 \quad \Rightarrow \quad h = \frac{\gamma_1 - \gamma_2}{\rho g a} \]
Griffith’s experiment

Reference state:

$$U_0 = \frac{\sigma^2}{2E} V$$

Introduce a crack:

$$\Delta U \sim -\frac{\sigma^2}{2E} a^2 t$$

Total energy change due to crack:

$$\Delta G = \Delta U + 2\gamma at = 2\gamma at - g \frac{\sigma^2}{2E} a^2 t$$
Critical condition

If $a < a^*$, the crack heals to reduce energy.

If $a > a^*$, crack grows.

Critical crack size: $\frac{\partial \Delta G}{\partial a} = 0 \quad \rightarrow \quad a^* = \frac{E\gamma}{g\sigma^2}$

Critical stress for a given crack: $\sigma_c = \sqrt{\frac{E\gamma}{ga}}$

Agree with Griffith’s experiments.

Deduced surface energy density on the order of 1J/m$^2$ (fantastic for glass!).
Effect of plastic deformation

• Applying Griffith’s energy approach to ductile materials (e.g., steels) leads to unreasonably big surface energy density, but $\sigma_c \sim c^{-1/2}$ still correct.

• Irwin’s idea: plastic deformation near the crack dissipates energy, need more energy to grow a crack!

• Fracture resistance include both surface energy and plastic energy dissipation.

• Resistance to create a crack of unit area:
  
  For glass, $u_p \ll \gamma$, $\Gamma \sim 2\gamma \sim 1-10$ J/m²
  
  For steels, $u_p \gg \gamma$, $\Gamma > 1000$ J/m²
Energy release rate

- Elastic consideration, ignore plastic deformation.
- Total elastic strain energy is a function of the displacement at the loading point and the area of crack surfaces.

\[ U = U(\Delta, A) \]
\[ dU = F d\Delta - G dA \]

Energy release rate: Elastic energy released by a unit area increase of crack with fixed displacement.
Energy release rate

For Griffith’s problem:

\[ U = U_0 - \frac{\pi \sigma^2}{2E} a^2 t \]

\[ G = -\left( \frac{\partial U}{\partial A} \right)_\Delta = \frac{\pi \sigma^2 a}{E} \]

In general:

\[ G = Y \frac{\sigma^2 a}{E} \]

\( Y \) = a dimensionless number depending on geometry and loading conditions, solved by elasticity.
Measure compliance to determine $G$

Determine compliance from load-displacement curve

$$C(A) = \frac{\Delta}{P}$$

Elastic strain energy:

$$U = \frac{1}{2} P\Delta = \frac{\Delta^2}{2C(A)}$$

Energy release rate:

$$G = -\left(\frac{\partial U}{\partial A}\right)_{\Delta} = \frac{\Delta^2}{2C^2} \left(\frac{\partial C}{\partial A}\right) = \frac{P^2}{2b} \frac{\partial C}{\partial a}$$
Double cantilever beam

Deflection of a cantilever beam of length $c$:

$$\Delta = \frac{Pc^3}{3EI}$$

Elastic strain energy in DCB:

$$U = 2 \cdot \frac{1}{2} P\Delta = \frac{P^2 c^3}{3EI}$$

Energy release rate:

$$G = -\left(\frac{\partial U}{\partial A}\right)_\Delta = \frac{12P^2 c^2}{EH^3 B^2}$$
Fracture criterion

- Driving force: elastic energy release ($G$)
- Fracture Resistance ($\Gamma$): surface energy and plastic energy dissipation.

Criterion for crack growth:

\[ G = \Gamma \]

\[ \Gamma = 2\gamma + u_p \]
Relation between $G$ and $K$

\[ G = \frac{K^2}{E} \]

\[ G_c = \frac{K_c^2}{E} \]

\[ \bar{E} = \begin{cases} \frac{E}{E} & \text{plane stress} \\ \frac{E}{1-v^2} & \text{plane strain} \end{cases} \]

- Can be used interchangeably.
- Energy approach is important and sometimes more convenient in fracture analysis.
Summary

• Energy approach in fracture mechanics
  – Elastic energy release rate (G)
  – Fracture resistance (Γ, toughness): surface energy plus plastic dissipation

• Determine G by compliance

• Equivalence of G and K