Poroelastic Effect on Fracture of Gels

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Hydrogel is Poroelastic

- **Nonlinear Poroelasticity**: large and reversible deformation of polymer network coupled with migration (diffusion) of solvent molecules, in response to mechanical, chemical, and other environmental stimuli (temperature, pH, light, etc)

- Applications: biomedical, soft machines...
Fracture Toughness of Gels

- Reported values of fracture toughness range from 1 to $10^4$ J/m$^2$
- Fracture mechanism may vary over different types of gels
- Potential rate dependence may relate to viscoelasticity or solvent diffusion (poroelasticity) or ...
Delayed Fracture of Gels

Bonn et al., 1998

Tang et al., 2017

Time to fracture (nucleation):

\[ \ln t_b \sim \sigma^{-2d+2} \]

A window of delayed fracture
Rate-dependent Fracture of Gels

Baumberger et al., 2006

Lefranc and Bouchaud, 2014
Energetics of Fracture (J-integrals)

- **Total energy flux** (remote loading)
- **Large-scale processes** (energy dissipation by plasticity, viscoelasticity, diffusion, etc)
- **Small-scale processes** (fracture)

• The elastic energy is stored and (partly) released upon fracture.
• The elastic energy release rate is the driving force for fracture.

\[
J_{\text{remote}} = J_{\text{elastic}} + J_{\text{inelastic}}
\]

\[
\Gamma_{\text{fracture}} = J_{\text{elastic}} = J_{\text{remote}} - J_{\text{inelastic}}
\]
Energy release rate for gels: a modified J-integral

- $J^*$ is path independent.
- The second form is more convenient for numerical calculations.
- A domain integral method can be used to calculate $J^*$.

$$J^* = \int_{S_l} \left( UN_1 - s_{iJ} N_j \frac{\partial x_i}{\partial X_1} \right) dS - \int_{V_0} \mu \frac{\partial C}{\partial X_1} dV$$

$$J^* = \int_{S_l} \left( \hat{U}N_1 - s_{iJ} N_j \frac{\partial x_i}{\partial X_1} \right) dS + \int_{V_0} \frac{\partial \mu}{\partial X_1} CdV$$

$$\hat{U}(\mathbf{F}, \mu) = U(\mathbf{F}, C) - \mu C$$

Proposed Fracture Criteria for Gels

- For a stationary pre-existing crack:
  \[ J^*(t) = \Gamma_0 \]  
  → Delayed fracture

- For steady-state crack growth:
  \[ J^*(\dot{a}) = \Gamma_{SS} \]  
  → Rate-dependent fracture

“Division of labor”:
- Calculate the crack-tip energy release rate
- Measure the *intrinsic* fracture toughness
Transient full-field finite element analysis

Time-dependent Energy Release Rate

\[ \frac{J^*}{J_0} = \Lambda \left( \frac{t}{\tau}, \frac{h}{a}, \nu \right) \]

\[ J_0 \sim \sigma_h^2 a/G \]

- The short-time limits are different for immersed and not-immersed specimens.
- The long-time limits are different for immersed specimens under displacement and load control.

Delayed Fracture by Poroelasticity

$$J^*(t) = J_0 \Lambda \left( \frac{t}{\tau} \right) = \Gamma$$

$$t_b = \frac{a^2}{D^*} f \left( \frac{J_0}{\Gamma}, \nu \right)$$

Critical condition for instantaneous fracture:

$$J^*(t \to 0^+) > \Gamma$$

Threshold for delayed fracture:

$$J^*(t \to \infty) > \Gamma$$

Same threshold but different critical loads for immersed and not-immersed specimens.

Steady-state crack growth model

- A semi-infinite crack in an infinitely long strip made of a linearly poroelastic material;
- Uniform vertical displacements are applied at the top and bottom, while the crack grows in a steady state.
- Ignore inertia for quasi-static crack growth.

Numerical Results

\( Pe = \dot{\alpha} h / D^* = 10, \quad \nu = 0.2414 \)

Poroelastic shielding: the crack-tip stress intensity factor is lower than the elastic case.

\[
\frac{K_{tip}}{K_e} = f(Pe, \nu) \quad \quad K_{tip} < K_e = 4 G \varepsilon_{\infty} \sqrt{h}
\]

The apparent energy release rate is greater than the intrinsic fracture energy (toughness) because energy dissipation by solvent diffusion around the crack tip.

Experimental implications

Measure intrinsic fracture toughness as a function of crack speed:

\[ \Gamma = J^* = 4Ge^2 h \Lambda \left( \frac{\dot{a}h}{D^*}, \nu \right) \]

- “velocity toughening”: the apparent fracture energy increases with crack speed.
- Effect of solvent viscosity: high viscosity -> low diffusivity -> low crack speed
- Effect of crack-tip soaking: immersed (wet) versus not-immersed (dry)

Solvent diffusion dissipates less energy under plane stress, opposite to the plastic dissipation in metals.
A poroelastic cohesive zone model

Rate-dependent fracture toughness

Solvent diffusion within the cohesive introduces addition toughening effect, leading to rate-dependent fracture.

Summary: Fracture Criteria for Gels

- For a stationary pre-existing crack:
  \[ J^*(t) = \Gamma_0 \]  
  \( \rightarrow \)  
  Delayed fracture

- For steady-state crack growth:
  \[ J^*(\dot{a}) = \Gamma_{SS} \]  
  \( \rightarrow \)  
  Rate-dependent fracture

- A poroelastic cohesive zone model predicts rate-dependent traction-separation relations and additional toughening.