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Trapped torsional vibrations in elastic plates

T. Knowles
Texzec, Inc., Round Rock, Texas 78681

M. K. Kang and R. Huang
Department of Aerospace Engineering and Engineering Mechanics,
University of Texas, Austin, Texas 78712

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We report observation and analysis of trapped torsional vibrations in elastic plates. Each trapping element consists of a circular mesa machined in cast aluminum plate, with an electromagnetic acoustic transducer used to generate oscillatory surface traction. Suitably applied traction induced torsional vibrations trapped in the mesa. The resonant frequencies, relative displacements and Q-values were measured, and an approximate theory was developed to analyze the trapping effect with good agreement between measurements and theory. It was found that these trapped torsional modes have Q-values exceeding 100 000 with pure in-plane motion, which is of practical importance for acoustic sensor applications. © *2005 American Institute of Physics.*
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Mechanical resonators with energy trapping typically have low losses and hence high quality factors (high-Q),[1] and are therefore sensitive to surface loading. An example of this is the thickness-shear mode quartz crystal microbalance (QCM),[2,3] which has found extensive applications from viscometers to biodetectors. Thickness-shear mode vibrations are confined under a thin metal electrode deposited on the crystal surface, which eliminates crystal edges and mounting structures as sources of energy loss.[4] The quartz plate can be regarded as an acoustic waveguide, with the electrode acting as a mass load. The mass reduces the waveguide cut-off frequency and results in a frequency band, within which at least one trapped resonance exists.[5] A well-known problem with the thickness-shear mode QCM is that in-plane shear motion intrinsically couples to out-of-plane flexural modes.[6,7] When in contact with or immersed in liquids, the out-of-plane motion generates compressional waves that reflect off the liquid surface and return to the crystal. This interference effect causes undesired depth-sensitive perturbations in the sensor response. Here we propose to use torsional modes to eliminate this effect.

We have discovered that torsional vibration modes can be trapped in elastic plates with circular regions of slightly thicker steps or with smooth convex contoured surfaces. Energy trapping has also been observed in elastic cylinders, where torsional vibration modes can be trapped within a stepped region of slightly larger diameter.[8] Torsional vibrations do not couple to flexural modes, have no out-of-plane motions, and hence no interference effect or liquid level sensitivity, allowing an improved response compared to the thickness-shear mode. This letter reports observation and analysis of trapped torsional modes in elastic plates.

The energy-trapping element here consists of a circular mesa as shown in Fig. 1. These elements were machined in a 6000 series cast aluminum plate, with the mesa perimeters defined by removing metal in the form of a moat. The plate thickness (~3 mm) is approximately one-half wavelength of the fundamental mode and the moat depths range from 2% to 7% of the plate thickness. Mesa diameter ranges from 2.5 cm to 4 cm. The moat was made wide enough (typically exceeding 2 or more wavelengths) so that the displacement, which decayed exponentially away from the mesa perimeter, was negligible at the outer edge of the moat and hence the mesa was isolated acoustically.

A noncontact electromagnetic acoustic transducer or EMAT was employed to set the mesa into motion. The particular set-up used in this study is illustrated in Fig. 2, and consists of a spiral coil and two permanent magnets. Oscillatory currents in the coil induce eddy currents in the metal plate, which generate desired oscillatory surface tractions in the presence of the vertical component of the permanent magnetic field. EMATs are not efficient electrical to mechanical energy converters, but they are well suited for this application because energy leakage is very small due to trapping and intrinsic losses in aluminum are low. The mesa was set into resonant vibration by applying a sinusoidal current pulse train to the coil. Upon termination of the excitation waveform, the displacement and hence the induced coil voltage amplitude decayed exponentially with time.

Torsional modes were excited when the traction force was applied in the circumferential direction. Resonant frequencies were determined by adjusting the pulse train frequency for peak initial amplitude as observed with an oscil-

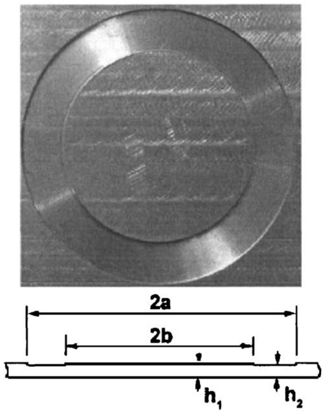

FIG. 1. The top view of a trapping element machined in a cast aluminum plate, and the schematic side view.

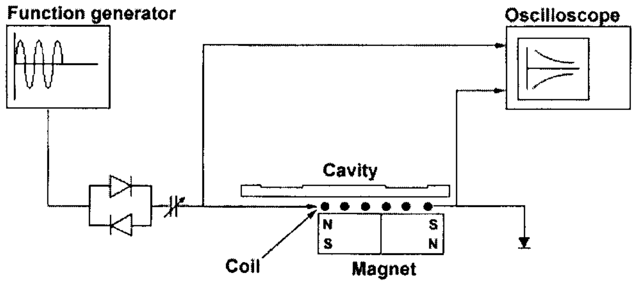

FIG. 2. Illustration of the experimental set-up.

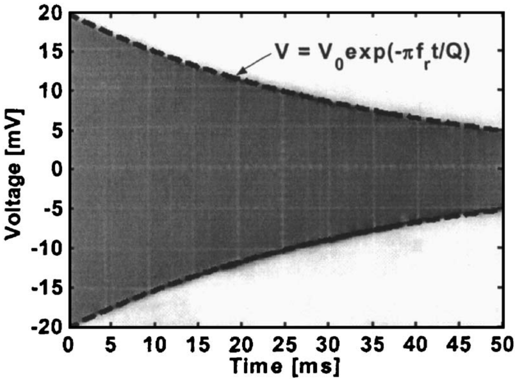

FIG. 4. The decay envelop at a resonance frequency. The exponential decay function with f_r=952 kHz and Q=108 000 is plotted as the dashed lines.

loscope across the EMAT coil. Figure 3 shows an example of the measured initial amplitude as a function of frequency, from which a resonance frequency at 923.1 kHz and a Q-value of 12 300 were determined. The Q-value can also be calculated from the corresponding decay envelope at the resonant frequency.[9] It was confirmed that the frequency and time domain methods of determining Q gave the same value. However, the frequency resolution of the equipment we had on hand limited us to relatively low Q. For this reason, a small section of absorbing tape on the mesa was used in this measurement to suppress Q. With the tape removed, and using the time domain method, Q-values greater than 100 000 were obtained. Figure 4 shows a decay envelop with a Q-value of 108 000.

Trapping of torsional modes can be understood as a result of the reduced cut-off frequency in the circular mesa. The cut-off frequency for the first-order torsional modes, $\omega_1=(\pi/h)\sqrt{\mu/\rho}$, depends on the plate thickness h, where μ is the shear modulus and ρ is the mass density. There exists a frequency gap between the cut-off frequencies in a stepped mesa and the outer region of the plate, within which the torsional resonances, if they exist, will be trapped. An approximate theory of stepped plates was developed to analyze the trapped torsional modes.[10] The resonance frequencies predicted by this theory are compared to the measured frequencies, as listed in Table I. The agreement is excellent except for sample B, where only two trapped modes were observed while three were predicted by the theory. It was noted that spurious modes were generated and not trapped when the step thickness was relatively large and the frequency was high. Both experiments and theory indicate that the existence of the trapped torsional modes depends on the thickness and diameter of the circular mesa. When both the thickness and the diameter were very small, no trapped mode could be detected. As the diameter and/or the thickness increases, the number of trapped modes also increases. Figure 5 shows a diagram for the number of trapped torsional modes of first order predicted by the theory, which may serve as a design guide. It is noted that, for a given step thickness, there is a minimum diameter for trapping to occur, and this was confirmed in experiments using mesas of varying diameter.

A series of tests were conducted to measure the surface motion at resonances in order to confirm the observation of trapped torsional modes. First, taking advantage of its noncontact property, the EMAT-induced traction was shifted along an arbitrary radial line of the mesa, and we observed that the initial voltage amplitude varied with one or more peaks. We surmized that the peaks occur at radial distances corresponding to displacement maxima of the mode contour, and this was confirmed through the use of an additional test coil to be described later. This effect provides a convenient method of locating displacement maxima, simply by translating the plate with respect to the magnet and the coil along the magnet junction and recording the distance from the mesa center at an amplitude peak. The measured radial distances of displacement maxima agree closely with the theoretical predictions of mode contours.

A stylus tipped with an absorbing material was then used to probe the plate surface. The vibration amplitude is very sensitive to surface energy loss because of the high Q. We observed a sharp minimum in the amplitude when the stylus touched the surface at locations with displacement maxima. When the stylus probe was moved along a set of radial lines

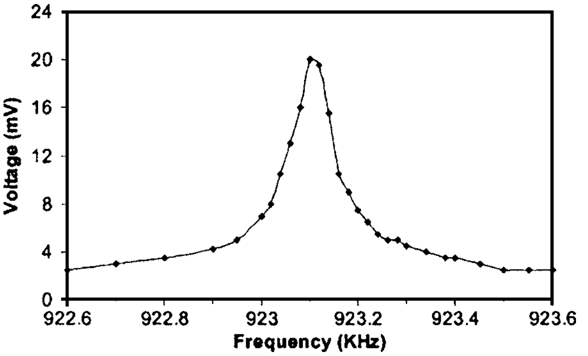

FIG. 3. Measured initial decay amplitude as a function of frequency, with a resonance at 923.1 kHz and a Q-value of 12 300 determined from the half width of the spectrum.

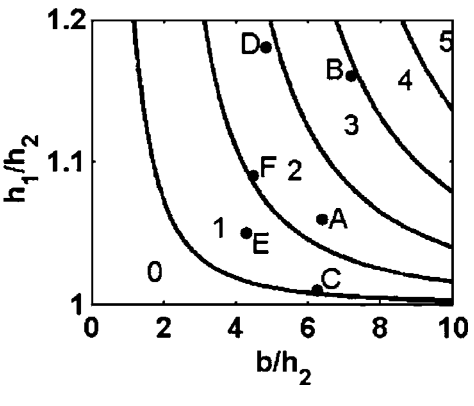

FIG. 5. A contour diagram for the number of first-order trapped torsional modes in circular mesas of various dimensions. The solid dots indicate the locations of the six samples listed in Table I.

TABLE I. Sample dimension and measured resonance frequencies in comparison with theoretical predictions. The shear modulus (26.37 GPa) was determined from the first resonance frequency of sample A. The mass density of the aluminum was taken to be 2700 kg/m^3.

Samples	A	B	C	D	E	F
2a(mm)	54.610	54.610	54.610	42.418	42.418	42.418
2b(mm)	37.668	37.846	37.719	25.908	25.654	25.654
h_1(mm)	3.1369	3.0683	3.0607	3.1679	3.1496	3.1242
h_2(mm)	2.9464	2.6416	3.0226	2.6797	2.9972	2.8702
Measured freq.	505.5/521.9	515.9/533.1/⋯	515.6	506.9/542.6	509.6	515.6/546.1
Theory	505.5/521.9	517.0/534.9/562.1	515.6	508.3/543.2	509.2	514.5/543.9

through 360°, the radial distances of the displacement maxima did not change, indicating radially symmetric motion which is characteristic of torsional modes. On the other hand, probing the surface outside the mesa has no effect on the amplitude, confirming that the motion is trapped within the mesa.

The absorbing stylus technique, though useful, cannot measure the direction and relative amplitudes of the displacement. To do this, a variety of methods can be used, the most convenient here being a pick-up coil. Vibration of the metal plate in the presence of a magnetic field will induce electrical currents normal to the displacement direction. A small pick-up coil with windings adjacent and parallel to the top surface of the mesa will have an induced voltage proportional to the current in the plate. The maximum voltage occurs when the coil windings are aligned along the current direction (normal to the displacement direction) and positioned at a displacement maximum. Different from thickness-shear modes where the displacement is in the radial direction, torsional modes have circumferential displacements in the plane of the plate. These modes were confirmed as the pick-up coil had maximum output when it was aligned in a radial direction (normal to the motion), and the amplitude and phase of the coil output were independent of the radial angle.

The pick-up coil is not sensitive to out-of-plane displacement. Thus the coil responses though suggestive do not definitively indicate pure in-plane motion. Additional confirmation was obtained by observing the effects of water depth. As discussed previously, thickness-shear mode resonators are sensitive to water depth due to coupling with out-of-plane displacements. The samples were placed in a fixture with the upper surface of the mesa exposed to water whose depth was varied. By changing the position of the EMAT coil with respect to the mesa, either thickness-shear or torsional modes were selected. For the thickness-shear mode, both the resonance frequency and the Q-value were sensitive to the water level as expected. For the torsional modes, the resonance frequency was independent of the water level, suggesting pure in-plane surface motion with negligible out-of-plane component. There was a fixed reduction of Q-value to approximately 7000 (at 953 kHz) in contact with water, independent of water depth.

In summary, we report observation of trapped torsional vibration in elastic plates with stepped circular mesas. Our tests confirmed that the motion was torsional and trapped within the mesa. The measured resonance frequencies agreed closely with theoretical predictions. A diagram was constructed to predict the number of trapped torsional modes in stepped plates. The torsional modes have very high Q-values with pure in-plane surface motion, which is particularly important for sensor applications in liquid environments and are potentially useful for other types of sensors as well as high performance resonators.

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