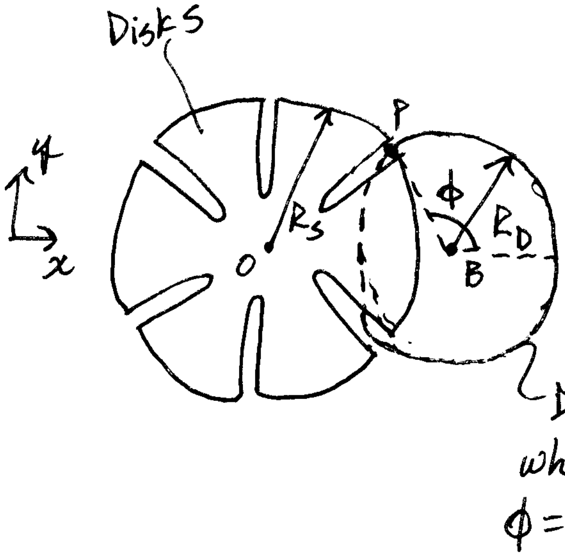
Example Problem



$R_S = \sqrt{3}\, R_D$

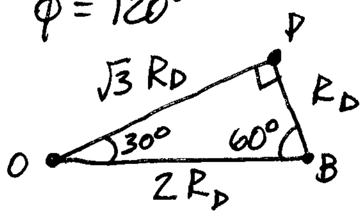
$R_D = 1.25 \text{ in}$

$\vec{\omega}_D = 8 \frac{\text{rad}}{\text{s}} \vec{k}$ (constant)

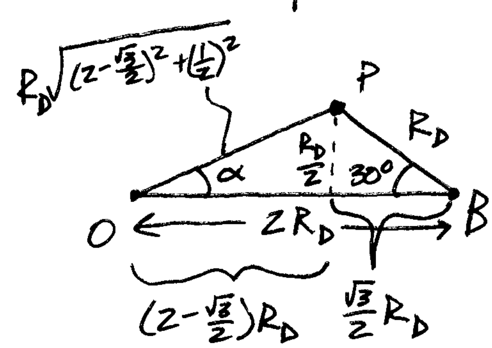
$\rightarrow \vec{\alpha}_D = 0$

Determine $\vec{\omega}_S$ and $\vec{\alpha}_S$ when $\phi = 150°$.

When $\phi = 120°$



When $\phi = 150°$



$\rightarrow \alpha = 23.794°$

$$\vec{v}_P = \vec{v}_B + \vec{\omega}_D \times \vec{r}_{P/B} \quad \leftarrow \text{P attached to D}$$

$$\vec{v}_P = 0 + 8\vec{k} \times R_D\left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$= 8R_D\left(-\frac{\sqrt{3}}{2}\vec{j} - \frac{1}{2}\vec{i}\right) = -5\vec{i} - 5\sqrt{3}\,\vec{j}$$

$$\vec{a}_P = \cancelto{0}{\vec{a}_B} + \cancelto{0}{\vec{\alpha}_D} \times \vec{r}_{P/B} + \vec{\omega}_D \times (\vec{\omega}_D \times \vec{r}_{P/B})$$

$$\therefore\ \vec{a}_P = 8\vec{k} \times (-5\vec{i} - 5\sqrt{3}\,\vec{j})$$

$$= 40\sqrt{3}\,\vec{i} - 40\vec{j}$$

Analyze $\vec{v}_P$ & $\vec{a}_P$ with respect to disk S.

$$\vec{v}_P = \cancel{\vec{v}_O}^{\,0} + \vec{v}_{P/\text{Body}\,S} + \vec{\omega}_S \times \vec{r}_{P/O}$$

$\vec{v}_{P/\text{Body}\,S}$ is in the direction of the slot.

i.e. $\vec{v}_{P/\text{Body}\,S} = v_{P/S}\,(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$

$$\therefore\ -5\vec{i} - 5\sqrt{3}\,\vec{j} = v_{P/S}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$

$$+\ \omega_S\,\vec{k} \times r_{P/O}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$

Note $r_{P/O} = R_D\sqrt{(2-\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = 1.549$ in.

$$\therefore\ -5 = v_{P/S}\cos\alpha - r_{P/O}\,\omega_S \sin\alpha$$

$$-5\sqrt{3} = v_{P/S}\sin\alpha + r_{P/O}\,\omega_S\cos\alpha$$

$$-5\cos\alpha = v_{P/S}\cos^2\alpha - r_{P/O}\,\omega_S \sin\alpha\cos\alpha$$

$$-5\sqrt{3}\sin\alpha = v_{P/S}\sin^2\alpha + r_{P/O}\,\omega_S\sin\alpha\cos\alpha$$

$$v_{P/S} = -5\cos\alpha - 5\sqrt{3}\sin\alpha = -8.069\ \tfrac{in}{s}$$

$$\therefore \omega_s = \frac{5 + v_{P/S}\cos\alpha}{r_{P/O}\sin\alpha} = -3.81\ \frac{rad}{s}$$

$$\boxed{\vec{\omega}_s = -3.81\vec{k}\ \frac{rad}{s}}$$

$$\vec{a}_P = \cancel{\vec{a}_O}^{\,0} + \vec{a}_{P/Body\,S} + 2\vec{\omega}_S \times \vec{v}_{P/Body\,S} + \vec{\alpha}_S \times \vec{r}_{P/O} + \vec{\omega}_S \times (\vec{\omega}_S \times \vec{r}_{P/O})$$

Since the slot is <u>straight</u> $\vec{a}_{P/Body\,S}$ can be written as $\vec{a}_{P/Body\,S} = a_{P/S}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$

$\therefore$

$$40\sqrt{3}\,\vec{i} - 40\vec{j} = a_{P/S}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$
$$+ 2\omega_S\vec{k} \times v_{P/S}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$
$$+ \alpha_S\vec{k} \times r_{P/O}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$
$$+ \omega_S\vec{k} \times (\omega_S\vec{k} \times r_{P/O}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j}))$$

$$= \vec{i}\left[a_{P/S}\cos\alpha - 2\omega_S \overset{v_{P/S}}{\wedge} \sin\alpha - r_{P/O}\alpha_S\sin\alpha - r_{P/O}\omega_S^2\cos\alpha\right]$$
$$+ \vec{j}\left[a_{P/S}\sin\alpha + 2\omega_S \overset{v_{P/S}}{\wedge} \cos\alpha + r_{P/O}\alpha_S\cos\alpha - r_{P/O}\omega_S^2\sin\alpha\right]$$

Solve for $a_{P/S}$ and α_S

$$40\sqrt{3}\cos\alpha - 40\sin\alpha = a_{P/S} - r_{P/O}\,\omega_S^2$$

$$\therefore \; a_{P/S} = 69.74 \ \frac{in}{s^2}$$

$$40\sqrt{3}\sin\alpha + 40\cos\alpha = -2\omega_S v_{P/S} - r_{P/O}\,\alpha_S$$

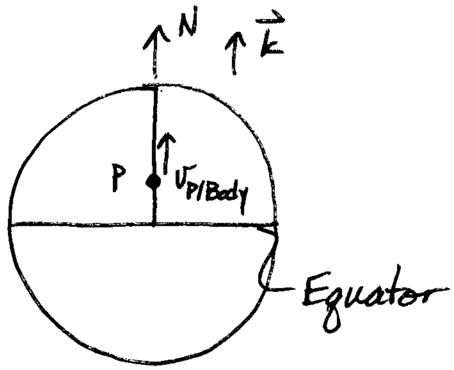
$$\therefore \; \alpha_S = -81.37 \ \frac{rad}{s^2}$$

$$\boxed{\alpha_S = -81.37 \vec{k} \ \frac{rad}{s^2}}$$

How the Coriolis Effect affects ocean currents

Recall: $\vec{v}_P = \vec{v}_o + \vec{\omega} \times \vec{r}_{P/O} + \vec{v}_{P/Body}$

$$\vec{a}_P = \vec{a}_o + \vec{\alpha} \times \vec{r}_{P/O} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O}) + \vec{a}_{P/Body} + 2\vec{\omega} \times \vec{v}_{P/Body}$$



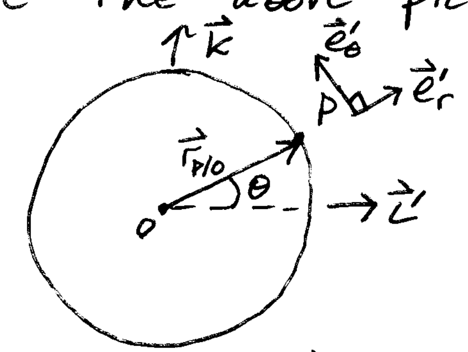
$$\vec{\omega} = \omega \vec{k}$$

$$\omega = \frac{2\pi}{24 \cdot 3600} = 7.27 \times 10^{-5} \ \frac{rad}{s}$$

$$\approx \frac{1}{13751} \ \frac{rad}{s}$$

$$\vec{\alpha} = 0$$

Assume we <u>force</u> P to move along 1 line of longitude.
Rotate the above picture by 90°



O is at the center of the earth, not on the equator

$\vec{i}'$, $\vec{j}'$ and $\vec{e}_r'$, $\vec{e}_\theta'$ rotate around with the earth

$$\vec{r}_{P/O} = R_e \vec{e}_r' = R_e (\cos\theta \, \vec{i}' + \sin\theta \, \vec{k})$$

$$\vec{v}_{P/Body} = v \, \vec{e}_\theta' = v(-\sin\theta \, \vec{i}' + \cos\theta \, \vec{k})$$

$$\vec{a}_{P/Body} = \frac{v^2}{R_e}(-\vec{e}_r') = -\frac{v^2}{R_e}(\cos\theta \, \vec{i}' + \sin\theta \, \vec{k})$$

$$\vec{a}_o = 0, \quad \vec{\alpha} = 0$$

Therefore: $\vec{a}_p = \omega\vec{k} \times [\omega\vec{k} \times R_e(\cos\theta\,\vec{i}' + \sin\theta\,\vec{k})]$

$$- \frac{v^2}{R_e}(\cos\theta\,\vec{i}' + \sin\theta\,\vec{k})$$

$$+ 2\omega\vec{k} \times v(-\sin\theta\,\vec{i}' + \cos\theta\,\vec{k})$$

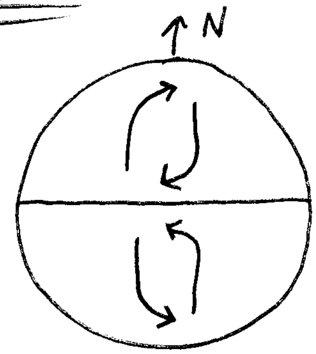
$$= \omega^2 R_e(-\cos\theta\,\vec{i}')$$

$$- \frac{v^2}{R_e}(\cos\theta\,\vec{i}' + \sin\theta\,\vec{k})$$

$$+ \underbrace{2\omega v(-\sin\theta\,\vec{j}')}_{\text{This is the only component in the } \vec{j}' \text{ direction.}}$$

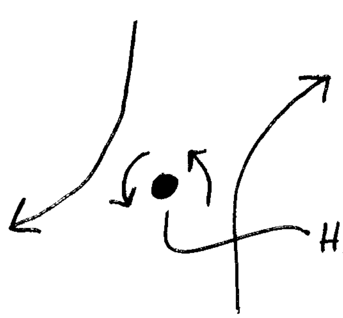
In order to support this acceleration in the $-\vec{j}'$ direction, we must apply a force to the particle in the $-\vec{j}'$ direction. In other words, if we want the particle to travel straight along a single line of longitude we must apply a lateral force towards the west. Note, if our particle was moving from the equator towards the ~~no~~ south pole the v would change sign and $\sin\theta$ would change sign and we would still need to apply a force to the west.

However, we are not there to apply forces to water currents to keep them along a single line of longitude. Therefore, any current traveling from the equator towards a pole will be deflected to the <u>east</u>. Currents traveling towards the equator will be deflected towards the <u>west</u>.



But why do hurricanes in the northern hemisphere rotate <u>counter</u>clockwise?

The eye of a hurricane is an atmospheric region of low pressure. Low pressure regions draw wind into them. Winds drawn in from the north are deflected to the west and those from the south are deflected to the east. This causes a <u>gear</u> effect on the air in the actual storm.



Hurricane.

Can the Coriolis effect affect the way that water goes down your sink?

Consider the magnitude of the effect.

$$a_{coriolis} = 2\omega v \sin\theta$$

$$= 2\left(7.27\times10^{-5}\ \tfrac{rad}{s}\right) v \sin\theta$$

Let's ignore $\sin\theta$ and take it to be its maximum value of 1. Also, in your sink a $1\ \frac{m}{s}$ current is relatively fast.

$$\therefore\ a_{coriolis} \approx 15\times10^{-5}\ \frac{m}{s^2}$$

This is approximately 5 orders of magnitude smaller than the acceleration due to gravity.

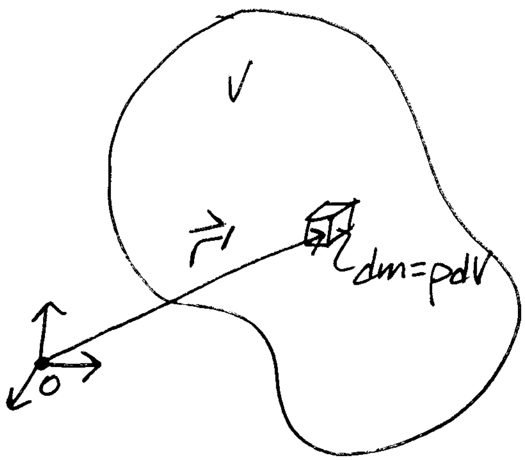
The fact is that this <u>can</u> affect how water goes down your sink, but <u>only</u> if your sink's shape and surface finish are practically flawless and the temperature of the surface is practically uniform. I assure you that the sink in your house is not this perfect and the Coriolis effect has nothing to do with whether water goes down clockwise or counter clockwise.

Kinetics of a Rigid Body

We will treat a rigid body as a collection of particles with the distances between them being fixed. Hence, our analysis for particles systems will be generalized for rigid bodies by replacing summations with integrations.

Recall for a system of particles:

$$\vec{r}_{cm} = \frac{1}{m}\sum_{i=1}^{n} m_i \vec{r}_i \quad \text{where} \quad m = \sum_{i=1}^{n} m_i$$



To generalize these equations we replace m_i with dm and $\sum_{i=1}^{n}$ with $\int_V$

$$\therefore \quad m = \int_V dm = \int_V \rho \, dV$$

then $\vec{r}_{cm} = \frac{1}{m}\int_V \vec{r}' \rho \, dV$

In component form

$$x_{cm} = \bar{x} = \frac{1}{m}\int_V x \rho \, dV$$

$$y_{cm} = \bar{y} = \frac{1}{m}\int_V y \rho \, dV$$

$$z_{cm} = \bar{z} = \frac{1}{m}\int_V z \rho \, dV$$

Notice that if the origin is placed exactly at the center of mass, then

$$\bar{x} = \bar{y} = \bar{z} = 0 \quad \text{or} \quad \vec{r}_{cm} = 0$$

This implies that $\int_V \vec{r}\,'\rho\, dV = 0$

$$\underbrace{\begin{aligned} \int_V x\rho\, dV &= 0 \\ \int_V y\rho\, dV &= 0 \\ \int_V z\rho\, dV &= 0 \end{aligned}}$$

All if the origin is located at the center of mass.

Now let's consider the laws of motion for the rigid body.

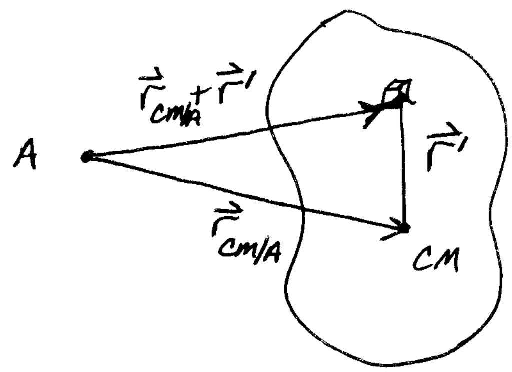
Recall $\quad \Sigma \vec{F}^{ext} = m\vec{a}_{cm}$ for a particle system.

This is unchanged for the rigid body.

$$\Sigma \vec{F} = m\vec{a}_{cm}$$

Also recall for the analysis of angular momentum we had

$$\Sigma \vec{M}_A^{ext} = \sum_{i=1}^{n} \frac{d}{dt}(\vec{r}_{i/A} \times m_i \vec{v}_i) + \vec{v}_A \times m \vec{v}_{cm}$$



Note that the "origin" for $\vec{r}\,'$ is located at the center of mass.

$$\rightarrow \int_V \vec{r}\,'\rho\, dV = 0$$

$$\rightarrow \vec{M}_A = \frac{d}{dt}\int_V (\vec{r}_{cm/A} + \vec{r}\,') \times \vec{v}\,\rho\, dV + \vec{v}_A \times m\vec{v}_{cm}$$

but $\vec{v} = \vec{v}_{cm} + \vec{\omega} \times \vec{r}\,'$

$$\therefore \int_V (\vec{r}_{cm/A} + \vec{r}\,') \times (\vec{v}_{cm} + \vec{\omega} \times \vec{r}\,')\rho\, dV$$

$$= \int_V \vec{r}_{cm/A} \times \vec{v}_{cm}\,\rho\, dV + \int_V \vec{r}_{cm/A} \times (\vec{\omega} \times \vec{r}\,')\rho\, dV$$

$$+ \int_V \vec{r}\,' \times \vec{v}_{cm}\,\rho\, dV + \int_V \vec{r}\,' \times (\vec{\omega} \times \vec{r}\,')\rho\, dV$$

$$= \vec{r}_{cm/A} \times \vec{v}_{cm} \int_V \rho\, dV + \vec{r}_{cm/A} \times \left(\vec{\omega} \times \int_V \vec{r}\,'\rho\, dV\right)$$

$$+ \int_V \vec{r}\,'\rho\, dV \times \vec{v}_{cm} + \int_V \vec{r}\,' \times (\vec{\omega} \times \vec{r}\,')\rho\, dV$$

but $\int_V \rho\, dV = m$ and $\int_V \vec{r}\,'\rho\, dV = 0$

$$\therefore\ \vec{M}_A = \frac{d}{dt}\underbrace{\left[\vec{r}_{cm/A}\times m\vec{v}_{cm} + \int_V \vec{r}\,'\times(\vec{\omega}\times\vec{r}\,')\rho\, dV\right]}_{\vec{h}_A \text{ for the rigid body}}$$

$$+\ \vec{v}_A \times m\vec{v}_{cm}$$

Let's consider $\int_V \vec{r}\,'\times(\vec{\omega}\times\vec{r}\,')\rho\, dV$ in general

$$\vec{r}\,' = x'\vec{i} + y'\vec{j} + z'\vec{k} \quad \text{(origin at CM)}$$

$$\vec{\omega} = \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k}$$

then $\int_V \vec{r}\,'\times(\vec{\omega}\times\vec{r}\,')\rho\, dV$

$$= \int_V \rho\Big[(x'\vec{i} + y'\vec{j} + z'\vec{k})$$
$$\times\left\{(\omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k})\times(x'\vec{i} + y'\vec{j} + z'\vec{k})\right\}\Big]dV$$

$$= \int_V \rho\Big[(x'\vec{i} + y'\vec{j} + z'\vec{k})$$
$$\times\left\{(\omega_y z' - \omega_z y')\vec{i} + (\omega_z x' - \omega_x z')\vec{j}\right.$$
$$\left. + (\omega_x y' - \omega_y x')\vec{k}\right\}\Big]dV$$

$$= \int_V \rho \Big[(\omega_x y'^2 - \omega_y x'y' - \omega_z x'z' + \omega_x z'^2)\vec{i}$$
$$+ (\omega_y z'^2 - \omega_z y'z' - \omega_x x'y' + \omega_y x'^2)\vec{j}$$
$$+ (\omega_z x'^2 - \omega_x x'z' - \omega_y y'z' + \omega_z y'^2)\vec{k}\Big] dV$$

Now define: $I_{xx} = \int_V \rho (y'^2 + z'^2)\, dV$

$$I_{xy} = -\int_V \rho\, x'y'\, dV$$
$$I_{xz} = -\int_V \rho\, x'z'\, dV$$
$$I_{yx} = -\int_V \rho\, x'y'\, dV = I_{xy}$$
$$I_{yy} = \int_V \rho (x'^2 + z'^2)\, dV$$
$$I_{yz} = -\int_V \rho\, y'z'\, dV$$
$$I_{zx} = -\int_V \rho\, x'z'\, dV = I_{xz}$$
$$I_{zy} = -\int_V \rho\, y'z'\, dV = I_{yz}$$
$$I_{zz} = \int_V \rho (x'^2 + y'^2)\, dV$$

These components form what is called the moment of inertia <u>tensor</u>.

$$\underset{\approx}{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

Then the angular momentum of the rigid body about its CM can be written as

$$\begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

Since we will only be dealing strictly with planar motion we won't have to worry about this complication.

For purely <u>planar</u> motion we can write

$$\vec{\omega} = \omega \vec{k}$$
$$\vec{r}\,' = r' \vec{e}_r + z' \vec{k}$$

then $\int_V \vec{r}\,' \times (\vec{\omega} \times \vec{r}\,') \rho \, dV = \int_V (r'\vec{e}_r + z'\vec{k}) \times [\omega \vec{k} \times (r'\vec{e}_r + z'\vec{k})] \rho \, dV$

$$= \int_V (r'\vec{e}_r + z'\vec{k}) \times (\omega r' \vec{e}_\theta) \rho \, dV$$

$$= \int_V (r'^2 \vec{k} - r'z' \vec{e}_r) \rho \omega \, dV$$

In addition to only planar motion we will also only analyze objects with geometry that is mirrored about the x-y plane, i.e. for every point x', y', z' there is also a point located at $x', y', -z'$ with exactly the same density.

Under these conditions $\int_V r'z' \rho \, dV = 0$

Then

$$\int_V \vec{r}' \times (\vec{\omega} \times \vec{r}') \rho \, dV = \underbrace{\int_V r'^2 \rho \, dV}_{I_{cm} \text{ or } \bar{I}} \omega \vec{k}$$

$$\therefore \quad \vec{h}_{cm} = I_{cm} \omega \vec{k}$$

or

$$\vec{M}_A = \frac{d}{dt} \left[\vec{r}_{cm/A} \times m \vec{v}_{cm} + I_{cm} \omega \vec{k} \right]$$

$$+ \vec{v}_A \times m \vec{v}_{cm}$$

Finally, if we take A to be a fixed point or the CM then

$$\frac{d}{dt}\left(\vec{r}_{cm/A} \times m\vec{v}_{cm}\right) = \underbrace{\vec{v}_{cm} \times m\vec{v}_{cm}}_{0} + \vec{r}_{cm/A} \times m\vec{a}_{cm}$$

$\vec{r}_{cm/A} = 0$ if $A = CM$ b/c $\vec{r}_{cm/cm} = 0$

also $\vec{v}_A \times m\vec{v}_{cm} = 0$ if A fixed or at CM.

$$\therefore \vec{M}_A = \frac{d}{dt}\left[\vec{r}_{cm/A} \times m\vec{v}_{cm} + I_{cm}\omega\vec{k}\right] + \vec{v}_A \times m\vec{v}_{cm}$$

$$\boxed{\vec{M}_A = \vec{r}_{cm/A} \times m\vec{a}_{cm} + I_{cm}\alpha\vec{k} \quad \text{for A fixed or CM}}$$

If A is fixed <u>and</u> attached to the body, the the body is rotating about A and we can write

$$\vec{a}_{cm} = \cancelto{0}{\vec{a}_A} + \vec{\alpha} \times \vec{r}_{cm/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{cm/A})$$

Then $\vec{r}_{cm/A} \times m\vec{a}_{cm} = \vec{r}_{cm/A} \times (\alpha\vec{k} \times \vec{r}_{cm/A})m$

$$+ \underbrace{\vec{r}_{cm/A} \times [\vec{\omega} \times (\vec{\omega} \times \vec{r}_{cm/A})]m}_{0}$$

$$= m\alpha\, r_{cm/A}^2\, \vec{k} = m\alpha d^2 \vec{k}$$

Where d is the distance between the CM and point A.

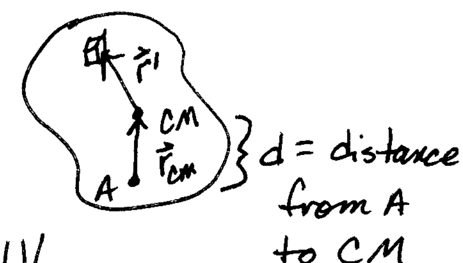
Then $\boxed{\vec{M}_A = (I_{cm} + md^2)\,\alpha\,\vec{k} \quad \text{if } A \text{ is fixed and attached to the body.}}$

Note that $I_{cm} + md^2$ is the moment of inertia of the body about an axis passing through point A.

i.e. $\quad I_A = I_{cm} + md^2$

then $\quad \vec{M}_A = I_A\,\alpha$

<u>Parallel Axis Theorem</u>



Thm: $I_A = I_{cm} + md^2$

Proof: $$I_A = \int_V \rho(\vec{r}_{cm} + \vec{r}\,')\cdot(\vec{r}_{cm} + \vec{r}\,')\,dV$$

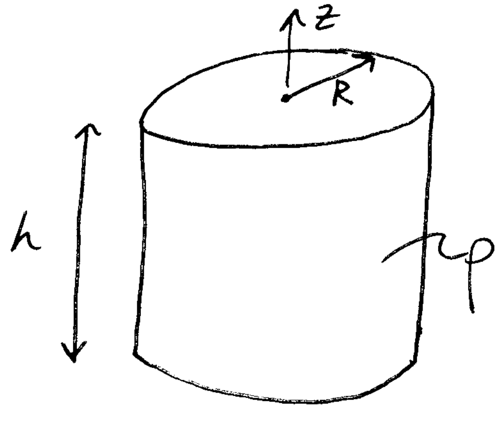
$$= \int_V \rho\left(d^2 + 2\vec{r}_{cm}\cdot\vec{r}\,' + r'^2\right)dV$$

$$= d^2\int_V \rho\,dV + 2\vec{r}_{cm}\cdot\int_V \rho\,\vec{r}\,'\,dV + \int_V \rho\,r'^2\,dV$$

$$= md^2 + 0 + I_{cm}$$ <u>QED</u>

Moment of inertia examples

Cylinder :



$V = \pi R^2 h$

$m = \rho \pi R^2 h$

ρ (uniform density)

Considering rotations about z-axis.

$$I_{cm} = \int_V \rho r'^2 dV$$

$$dV = r' dr' d\theta dz \quad , \quad \begin{matrix} 0 \le r' \le R \\ 0 \le \theta \le 2\pi \\ -\frac{h}{2} \le z \le \frac{h}{2} \end{matrix}$$

Origin must be located at cm.

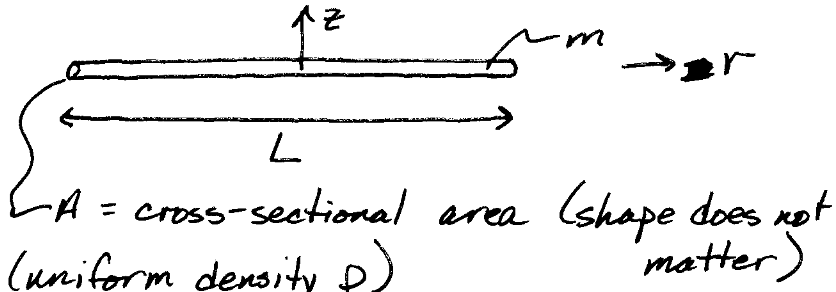
$$I_{cm} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^{2\pi} \int_0^R \rho \overbrace{r'^2 \, r'}^{r'^3} dr' d\theta dz$$

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^{2\pi} \underbrace{\rho \frac{1}{4} r'^4 \Big]_0^R}_{\frac{\rho}{4} R^4} d\theta dz$$

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\rho}{4} R^4 \underbrace{\theta \Big]_0^{2\pi}}_{2\pi} dz$$

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\rho\pi}{2} R^4 dz = \frac{\rho\pi}{2} R^4 z \Big]_{-\frac{h}{2}}^{h/2} = \frac{1}{2} R^2 \overbrace{\rho \pi R^2 h}^{m} = \frac{1}{2} m R^2$$

Slender Rod:



$V = LA$

$m = \rho L A$

$$I_{cm} = \int_V \rho r'^2 dV$$

$dV = A\,dr' \qquad -\frac{L}{2} \le r' \le \frac{L}{2}$

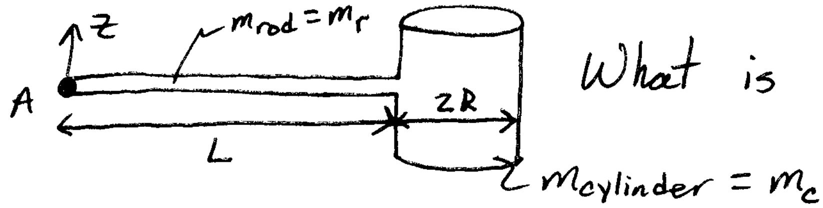
$$I_{cm} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \rho A r'^2 dr'$$

$$= \rho A \frac{1}{3} r'^3 \Big]_{-\frac{L}{2}}^{L/2} = \frac{\rho A}{3}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \frac{\rho A}{3}\frac{L^3}{4} = \frac{L^2}{12}\underbrace{\rho A L}_{m}$$

$$I_{cm} = \frac{1}{12} m L^2$$

Application of the parallel axis theorem.



What is I_A?

$$I_A = I_A^{Rod} + I_A^{Cylinder}$$

$$I_A^{Rod} = I_{cm}^{Rod} + m_{rod} d^2$$

$$= \frac{1}{12} m_r L^2 + m_r \left(\frac{L}{2}\right)^2$$

$$\boxed{I_A^{Rod} = \frac{1}{12} m_r L^2 + \frac{1}{4} m_r L^2 = \frac{1}{3} m_r L^2}$$

$$I_A^{cylider} = I_{cm}^{cylinder} + m_{cylinder} d^2$$

$$= \frac{1}{2} m_c R^2 + m_c (L+R)^2$$

$$= \frac{1}{2} m_c R^2 + m_c L^2 + m_c R^2 + 2 m_c L R$$

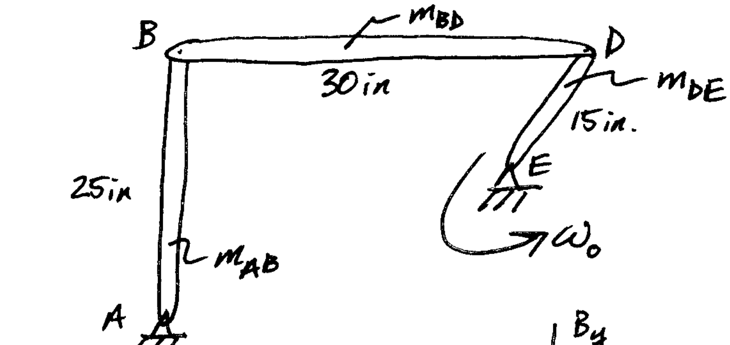
$$\boxed{I_A^{cylinder} = \frac{3}{2} m_c R^2 + m_c L^2 + 2 m_c L R}$$

Note a definition : $I_A = m k_A^2$

k_A = radius of gyration of the body about an axis passing through A.

i.e. $k_A = \sqrt{\frac{I_A}{m}}$

Analysis of Forces from Example Problem 16.67

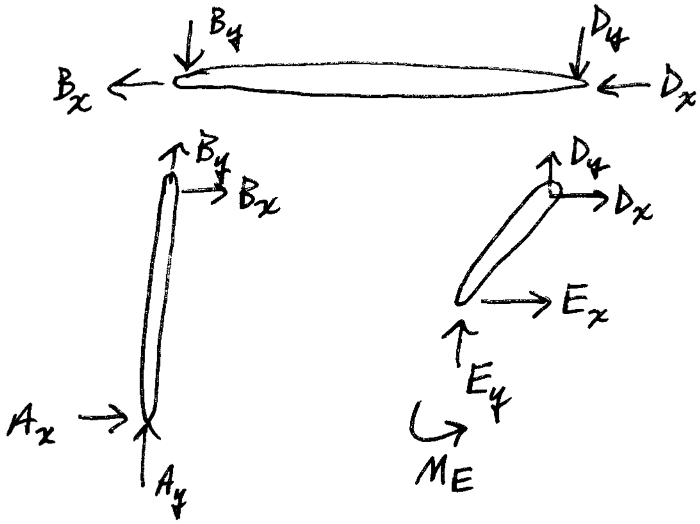


$\theta = 35°$

$\omega_0 = 4 \frac{rad}{s}$ & constant

Neglect gravity

FBDs :



Moments of Inertia:
$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$\bar{I}_{BD} = \frac{1}{12} m_{BD} L_{BD}^2$$
$$\bar{I}_{DE} = \frac{1}{12} m_{DE} L_{DE}^2$$

We also know: ω_{DE}, $\alpha_{DE} = 0$, ω_{BD}, α_{BD}, ω_{AB}, α_{AB} from our previous analysis.

Kinetics: Bar DE: $\sum \vec{F} = m\vec{a}_{cm} \rightarrow$
$$E_x + D_x = m_{DE}\, \bar{a}_{DE,x}$$
$$E_y + D_y = m_{DE}\, \bar{a}_{DE,y}$$

$$\sum \vec{M}^{cm} = \bar{I}\alpha \rightarrow M_E + (D_y - E_y)\frac{L_{DE}}{2}\sin 35° + (E_x - D_x)\frac{L_{DE}}{2}\cos 35° = \frac{1}{12} m_{DE} L_{DE}^2 \cancel{\alpha_{DE}}^{\,0}$$

B

Bar BD: $\sum \vec{F} = m\vec{a}_{cm} \Rightarrow -D_x - B_x = m_{BD}\,\bar{a}_{BD,x}$

$$-D_y - B_y = m_{BD}\,\bar{a}_{BD,y}$$

$$\sum \vec{M}^{cm} = \bar{I}\alpha \rightarrow (B_y - D_y)\frac{L_{BD}}{2} = \frac{1}{12} m_{BD} L_{BD}^2\,\alpha_{BD}$$

Bar AB: $\sum \vec{F} = m\vec{a}_{cm} \rightarrow A_x + B_x = m_{AB}\,\bar{a}_{AB,x}$

$$A_y + B_y = m_{AB}\,\bar{a}_{AB,y}$$

$$\sum \vec{M}^{cm} = \bar{I}\alpha \rightarrow (A_x - B_x)\frac{L_{AB}}{2} = \frac{1}{12} m_{AB} L_{AB}^2\,\alpha_{AB}$$

Unknowns: $A_x, A_y, B_x, B_y, D_x, D_y, E_x, E_y, M_E$
$\bar{a}_{AB,x}, \bar{a}_{AB,y}, \bar{a}_{BD,x}, \bar{a}_{BD,y}, \bar{a}_{DE,x}, \bar{a}_{DE,y}$

15 unknowns & 9 kinetics equations

But: $\bar{a}_{DE,x}\vec{i} + \bar{a}_{DE,y}\vec{j} = \cancel{\vec{a}_E}^{\,0} + \vec{\alpha}_{DE} \times \underbrace{\vec{r}_{CM/E}}^{\text{position of CM of DE w.r.t. E}} + \vec{\omega}_{DE} \times (\vec{\omega}_{DE} \times \vec{r}_{CM/E})$

$$\bar{a}_{AB,x}\vec{i} + \bar{a}_{AB,y}\vec{j} = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha}_{AB} \times \underbrace{\vec{r}_{CM/A}}_{\text{position of CM of AB w.r.t. A}} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{CM/A})$$

$$\bar{a}_{BD,x}\vec{i} + \bar{a}_{BD,y}\vec{j} = \vec{a}_B + \vec{\alpha}_{BD} \times \underbrace{\vec{r}_{CM/B}}_{\text{pos. of CM of BD w.r.t. B}} + \vec{\omega}_{BD} \times (\vec{\omega}_{BD} \times \vec{r}_{CM/B})$$

$$\underbrace{\text{with } \vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})}$$

8 more scalar Equations and 2 more unknowns, $a_{B,x}, a_{By}$ for a total of 17 Eqs. & 17 unknowns.