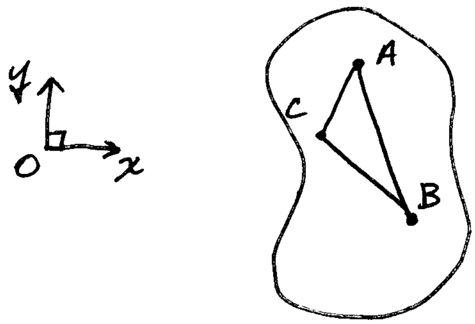
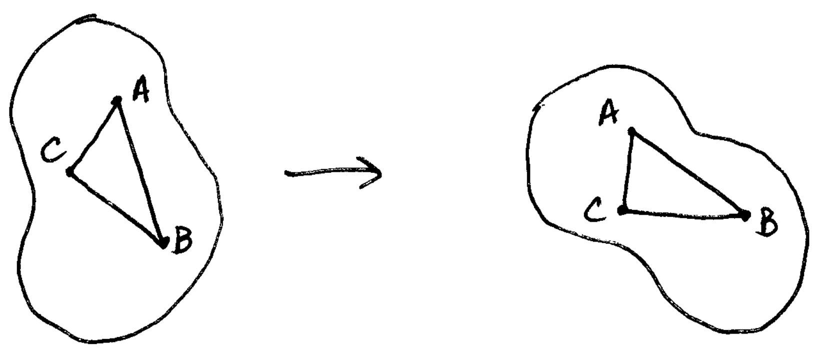
Rigid Body Dynamics

Kinematics

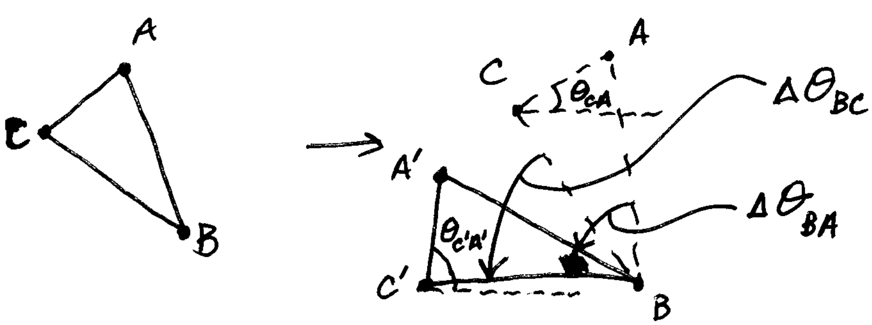


Within a rigid body the length of any line element connecting any two points in the rigid body remains unchanged for all time.

Furthermore, consider a planar rotation of the rigid body above.

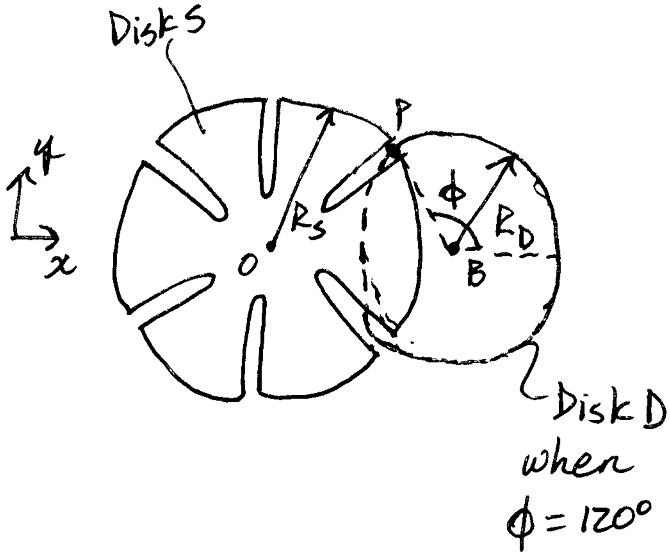


Consider the changes in angles with respect to the fixed coordinate system.



Note : $\Delta\theta_{BA} = \Delta\theta_{BC} = \theta_{C'A'} - \theta_{CA} = \Delta\theta_{CA}$

Example Problem



$R_S = \sqrt{3}\, R_D$

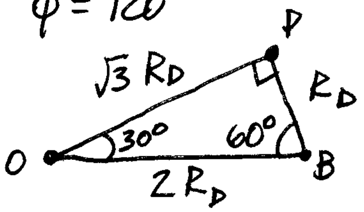
$R_D = 1.25\ in$

$\vec{\omega}_D = 8 \frac{rad}{s} \vec{k}$ (constant)

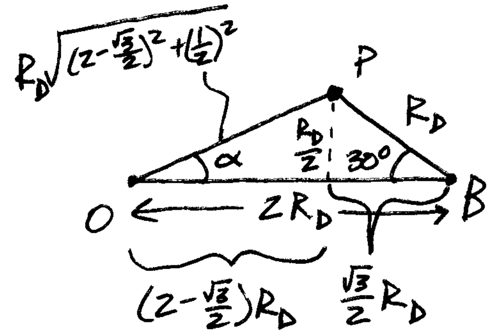
$\rightarrow \vec{\alpha}_D = 0$

Determine $\vec{\omega}_S$ and $\vec{\alpha}_S$ when $\phi = 150°$.

When $\phi = 120°$



When $\phi = 150°$



$\rightarrow \alpha = 23.794°$

$$\vec{v}_P = \vec{v}_B + \vec{\omega}_D \times \vec{r}_{P/B} \quad \leftarrow \text{P attached to D}$$

$$\vec{v}_P = 0 + 8\vec{k} \times R_D\left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$= 8R_D\left(-\frac{\sqrt{3}}{2}\vec{j} - \frac{1}{2}\vec{i}\right) = -5\vec{i} - 5\sqrt{3}\,\vec{j}$$

$$\vec{a}_P = \cancel{\vec{a}_B}^{\,0} + \cancel{\vec{\alpha}_D}^{\,0} \times \vec{r}_{P/B} + \vec{\omega}_D \times (\vec{\omega}_D \times \vec{r}_{P/B})$$

$$\therefore\ \vec{a}_P = 8\vec{k} \times (-5\vec{i} - 5\sqrt{3}\,\vec{j})$$

$$= 40\sqrt{3}\,\vec{i} - 40\vec{j}$$

Analyze $\vec{v}_P$ & $\vec{a}_P$ with respect to disk S.

$$\vec{v}_P = \cancelto{0}{\vec{v}_O} + \vec{v}_{P/Body\,S} + \vec{\omega}_S \times \vec{r}_{P/O}$$

$\vec{v}_{P/Body\,S}$ is in the direction of the slot.

i.e. $\vec{v}_{P/Body\,S} = v_{P/S}\,(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$

$$\therefore\ -5\vec{i} - 5\sqrt{3}\,\vec{j} = v_{P/S}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$

$$+\ \omega_S\,\vec{k} \times r_{P/O}(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$

Note $r_{P/O} = R_D\sqrt{(2-\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = 1.549$ in.

$$\therefore\ -5 = v_{P/S}\cos\alpha - r_{P/O}\,\omega_S \sin\alpha$$

$$-5\sqrt{3} = v_{P/S}\sin\alpha + r_{P/O}\,\omega_S\cos\alpha$$

$$-5\cos\alpha = v_{P/S}\cos^2\alpha - r_{P/O}\,\omega_S\sin\alpha\cos\alpha$$

$$-5\sqrt{3}\sin\alpha = v_{P/S}\sin^2\alpha + r_{P/O}\,\omega_S\sin\alpha\cos\alpha$$

$$v_{P/S} = -5\cos\alpha - 5\sqrt{3}\sin\alpha = -8.069\ \tfrac{in}{s}$$

$$\therefore \omega_S = \frac{5 + v_{P/S}\cos\alpha}{r_{P/O}\sin\alpha} = -3.81 \ \frac{rad}{s}$$

$$\boxed{\vec{\omega}_S = -3.81\vec{k} \ \frac{rad}{s}}$$

$$\vec{a}_P = \cancel{\vec{a}_O}^{\,0} + \vec{a}_{P/Body\,S} + 2\vec{\omega}_S \times \vec{v}_{P/Body\,S} + \vec{\alpha}_S \times \vec{r}_{P/O} + \vec{\omega}_S \times (\vec{\omega}_S \times \vec{r}_{P/O})$$

Since the slot is straight $\vec{a}_{P/Body\,S}$ can be written as $\vec{a}_{P/Body\,S} = a_{P/S}(\cos\alpha\vec{i} + \sin\alpha\vec{j})$

$\therefore$

$$40\sqrt{3}\,\vec{i} - 40\vec{j} = a_{P/S}(\cos\alpha\vec{i} + \sin\alpha\vec{j})$$
$$+ 2\omega_S\vec{k} \times v_{P/S}(\cos\alpha\vec{i} + \sin\alpha\vec{j})$$
$$+ \alpha_S\vec{k} \times r_{P/O}(\cos\alpha\vec{i} + \sin\alpha\vec{j})$$
$$+ \omega_S\vec{k} \times (\omega_S\vec{k} \times r_{P/O}(\cos\alpha\vec{i} + \sin\alpha\vec{j}))$$
$$= \vec{i}\left[a_{P/S}\cos\alpha - 2\omega_S \overset{v_{P/S}}{\wedge} \sin\alpha - r_{P/O}\alpha_S\sin\alpha - r_{P/O}\omega_S^2\cos\alpha\right]$$
$$+ \vec{j}\left[a_{P/S}\sin\alpha + 2\omega_S \overset{v_{P/S}}{\wedge} \cos\alpha + r_{P/O}\alpha_S\cos\alpha - r_{P/O}\omega_S^2\sin\alpha\right]$$

Solve for $a_{P/S}$ and α_S

$$40\sqrt{3}\cos\alpha - 40\sin\alpha = a_{P/S} - r_{P/O}\,\omega_S^2$$

$$\therefore \quad a_{P/S} = 69.74 \ \frac{in}{s^2}$$

$$40\sqrt{3}\sin\alpha + 40\cos\alpha = -2\omega_S v_{P/S} - r_{P/O}\,\alpha_S$$

$$\therefore \quad \alpha_S = -81.37 \ \frac{rad}{s^2}$$

$$\boxed{\alpha_S = -81.37\,\vec{k} \ \frac{rad}{s^2}}$$

Hence, for an in plane rotation the change in angle about <u>any</u> point is the same for the entire rigid body.

We showed in class, by considering 3 90° rotations of a book in different orders, that arbitrary changes in angle about the x, y and z axes do not add together like components of a vector, i.e.

$$\times \quad \overrightarrow{\Delta\theta} = \Delta\theta_x \vec{i} + \Delta\theta_y \vec{j} + \Delta\theta_z \vec{k} \quad \times \text{ Wrong}$$

is a meaningless equation because $\overrightarrow{\Delta\theta}$ cannot be represented as a vector.

However, if $\Delta\theta_x$, $\Delta\theta_y$ and $\Delta\theta_z$ are very small in comparison to 1, rigorously in the limit as $\Delta\theta_x \to 0$, $\Delta\theta_y \to 0$ and $\Delta\theta_z \to 0$, then and only then can $\overrightarrow{d\theta}$ be represented as a vector. i.e.

$$\overrightarrow{d\theta} = d\theta_x \vec{i} + d\theta_y \vec{j} + d\theta_z \vec{k} \quad \text{(differential } d\theta\text{s)}$$

for very small $d\theta$.

Then, derivative quantities like $\frac{d\vec{\theta}}{dt}$ and $\frac{d^2\vec{\theta}}{dt^2}$ are vector quantities.

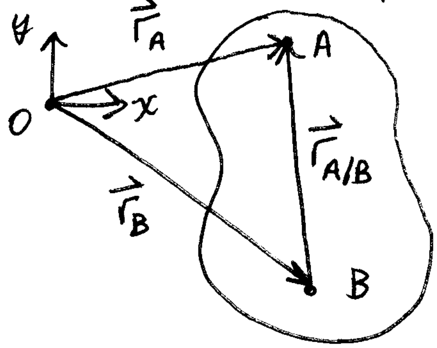
We call $\frac{d\vec{\theta}}{dt} = \vec{\omega} \equiv$ angular velocity

and $\frac{d^2\vec{\theta}}{dt^2} = \frac{d\vec{\omega}}{dt} = \vec{\alpha} \equiv$ angular acceleration

These quantities, $\vec{\omega}$ and $\vec{\alpha}$, are the same at an instant in time, about <u>any</u> point in the same rigid body.

Unlike a point mass whose position can be specified by 3 coordinates, the number of degrees of freedom for a rigid body is 6! Generally, these 6 degrees of freedom can be thought of as 3 linear coordinates and 3 angular orientations, but there are many other possible interpretations as well.

Relative Velocity and Acceleration



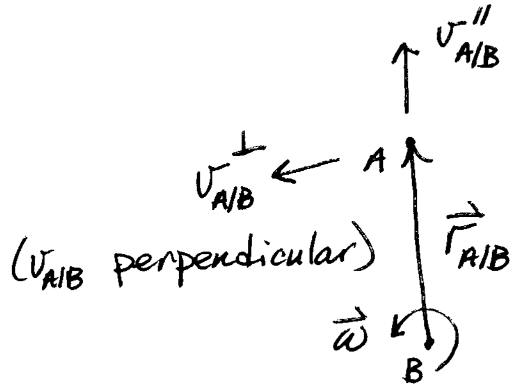
$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\vec{v}_A = \frac{d\vec{r}_A}{dt} = \underbrace{\frac{d\vec{r}_B}{dt}}_{\vec{v}_B} + \underbrace{\frac{d\vec{r}_{A/B}}{dt}}_{\vec{v}_{A/B}}$$

$$\vec{a}_A = \frac{d\vec{v}_A}{dt} = \underbrace{\frac{d\vec{v}_B}{dt}}_{\vec{a}_B} + \underbrace{\frac{d\vec{v}_{A/B}}{dt}}_{\vec{a}_{A/B}}$$

Consider $\vec{v}_{A/B} = \dfrac{d\vec{r}_{A/B}}{dt}$

We can always break this vector into components parallel and perdendicular to $\vec{r}_{A/B}$.



Note that $v_{A/B}^{\parallel}$ tends to increase the length between points A and B. However, from the definition of a rigid body we know that the distance between A and B must remain fixed for all time. Therefore $v_{A/B}^{\parallel} = 0$.

Now consider $v_{A/B}^{\perp}$. Assume that $\vec{\omega}$ is as shown so that AB is rotating CCW in the plane of the page. Then, applying scalar methods, we can see that $v_{A/B}^{\perp} = \omega r_{A/B}$.

In general we would like to make this a vector equation. Noting that $\vec{v}_{A/B}$ is perpendicular to $\vec{r}_{A/B}$ and both of these are perpendicular to $\vec{\omega}$, we can write

$$\boxed{\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}}$$

** $\frac{d\vec{r}_{A/B}}{dt} \neq 0$ → i.e. change in direction

$\frac{dr_{A/B}}{dt} = 0$ → i.e. no change in length AB

You should convince yourself that this is the appropriate order for the cross product.

Back to our relative velocity equation.

$$\boxed{\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}}$$

Then $$\frac{d\vec{v}_{A/B}}{dt} = \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{A/B}\right)$$

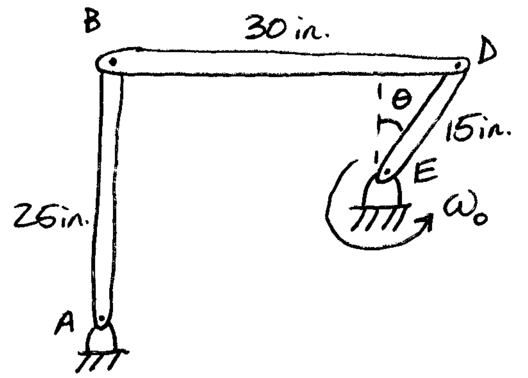
$$= \underbrace{\frac{d\vec{\omega}}{dt}}_{\vec{\alpha}} \times \vec{r}_{A/B} + \vec{\omega} \times \underbrace{\frac{d\vec{r}_{A/B}}{dt}}_{\vec{\omega} \times \vec{r}_{A/B}}$$

$$\boxed{\begin{aligned} \vec{a}_{A/B} &= \frac{d\vec{v}_{A/B}}{dt} = \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B}) \\ \vec{a}_A &= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B}) \end{aligned}}$$

Note that the parentheses are important b/c the order of × is important.

Note that $\vec{\alpha} \times \vec{r}_{A/B}$ is perpendicular to $\vec{r}_{A/B}$ and $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B})$ is in the opposite direction as $\vec{r}_{A/B}$.

Example Problem 16.67



$\theta = 35^\circ$

$\omega_0 = 4 \frac{\text{rad}}{\text{s}}$ CCW and constant

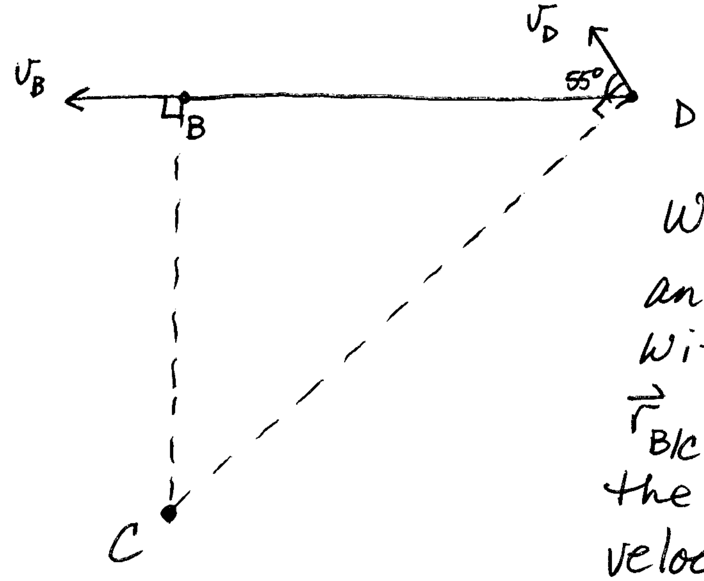
Determine α_{BD} and α_{AB} in the position shown.

Start with velocities and work around from E to A.

But first an aside. The instantaneous center of velocity of a rigid body is the point in the "rigid body extended" that has zero velocity at the instant in time under consideration. This point need not lie in the actual rigid body.

For bars DE and AB the instant center of velocity is obvious. It is the fixed point E for bar DE and the fixed point A for bar AB.

What about bar BD?



We know $\vec{v}_B \perp$ to $\vec{r}_{B/A}$ and $\vec{v}_D \perp$ to $\vec{r}_{D/E}$ along with $\vec{v}_B$ and $\vec{v}_D \perp$ to $\vec{r}_{B/C}$ and $\vec{r}_{D/C}$ where C is the instant center of velocity for bar BD.

Back to our problem.

$$\vec{v}_D = \vec{v}_E + \vec{\omega}_{DE} \times \vec{r}_{D/E}$$

$$v_{Dx}\vec{i} + v_{Dy}\vec{j} = 0 + \underbrace{\omega_{DE}}_{= \omega_0 = +4 \frac{rad}{s}}\vec{k} \times (15 \sin 35° \vec{i} + 15 \cos 35° \vec{j})$$

$$v_{Dx}\vec{i} + v_{Dy}\vec{j} = 60 \sin 35° \vec{j} - 60 \cos 35° \vec{i}$$

$$\therefore \quad v_{Dx} = -60 \cos 35°$$
$$v_{Dy} = 60 \sin 35°$$

Then to bar BD

$$\vec{v}_B = \vec{v}_D + \vec{\omega}_{BD} \times \vec{r}_{B/D}$$

$$v_{Bx}\vec{i} + 0\vec{j} = -60 \cos 35° \vec{i} + 60 \sin 35° \vec{j} + \underbrace{\omega_{BD}\vec{k} \times -30\vec{i}}_{-30\,\omega_{BD}\vec{j}}$$

we know this from looking at bar AB

$$\therefore \quad v_{Bx} = -60\cos 35^\circ$$

$$\boxed{\omega_{BD} = 2\sin 35^\circ = 1.147 \ \tfrac{rad}{s} \ \text{CCW}}$$

Bar AB

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$-60\cos 35^\circ \vec{i} = 0 + \underbrace{\omega_{AB}\vec{k} \times 25\vec{j}}_{-25\,\omega_{AB}\vec{i}}$$

$$\therefore \boxed{\omega_{AB} = \frac{60}{25}\cos 35^\circ = 1.966 \ \tfrac{rad}{s} \ \text{CCW}}$$

Now for acceleration. Bar DE.

$$\vec{a}_D = \cancel{\vec{a}_E}^{\,0} + \underbrace{\cancel{\vec{\alpha}_{DE}}}_{\to\, 0 \text{ b/c } \vec{\omega}_{DE} = \text{constant}} \times \vec{r}_{D/E} + \vec{\omega}_{DE} \times (\vec{\omega}_{DE} \times \vec{r}_{D/E})$$

$$a_{Dx}\vec{i} + a_{Dy}\vec{j} = 4\vec{k} \times (4\vec{k} \times (15\sin 35^\circ \vec{i} + 15\cos 35^\circ \vec{j}))$$

$$a_{Dx}\vec{i} + a_{Dy}\vec{j} = -240(\sin 35^\circ \vec{i} + \cos 35^\circ \vec{j})$$

$$\therefore \quad a_{Dx} = -240\sin 35^\circ$$
$$a_{Dy} = -240\cos 35^\circ$$

Bar BD

$$\vec{a}_B = \vec{a}_D + \vec{\alpha}_{BD} \times \vec{r}_{B/D} + \vec{\omega}_{BD} \times (\vec{\omega}_{BD} \times \vec{r}_{B/D})$$

$$a_{Bx}\vec{i} + a_{By}\vec{j} = -240(\sin 35^\circ \vec{i} + \cos 35^\circ \vec{j}) + \alpha_{BD}\vec{k} \times -30\vec{i} + \omega_{BD}\vec{k} \times (\omega_{BD}\vec{k} \times -30\vec{i})$$

$$= -240(\sin 35^\circ \vec{i} + \cos 35^\circ \vec{j}) - 30\alpha_{BD}\vec{j} + 30\underbrace{\omega_{BD}^2}_{4\sin^2 35^\circ}\vec{i}$$

<u>Bar AB</u>

$$\vec{a}_B = \cancel{\vec{a}_A}^{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})$$

$$a_{Bx}\vec{i} + a_{By}\vec{j} = \alpha_{AB}\vec{k} \times 25\vec{j} + \omega_{AB}\vec{k} \times (\omega_{AB}\vec{k} \times 25\vec{j})$$

$$= -25\alpha_{AB}\vec{i} - 25\underbrace{\omega_{AB}^2}_{\left(\frac{60}{25}\right)^2 \cos^2 35^\circ}\vec{j}$$

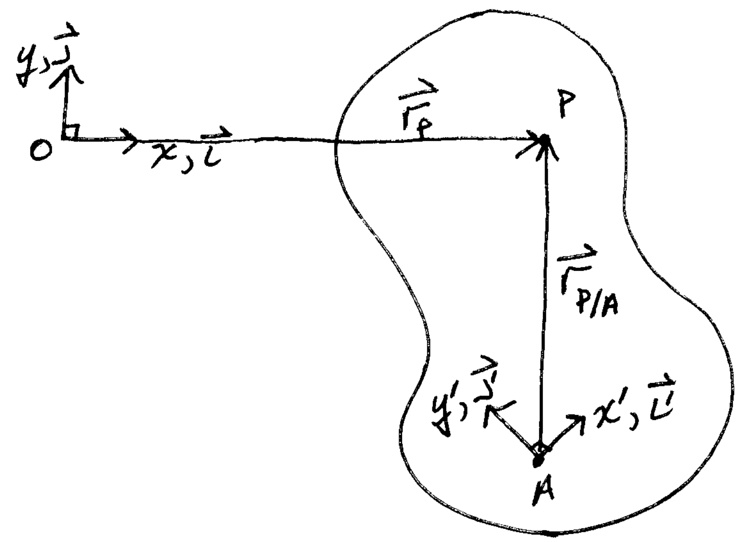
$$\therefore\ a_{Bx} = -25\alpha_{AB} = -240\sin 35^\circ + 120\sin^2 35^\circ$$

$$a_{By} = -\frac{3600}{25}\cos^2 35^\circ = -240\cos 35^\circ - 30\alpha_{BD}$$

$$\therefore\ \boxed{\begin{aligned} \alpha_{AB} &= 3.927\ \tfrac{\text{rad}}{s^2}\ \text{CCW} \\ \alpha_{BD} &= -3.332\ \tfrac{\text{rad}}{s^2}\ \text{CCW or } 3.332\ \tfrac{\text{rad}}{s^2}\ \text{CW} \end{aligned}}$$

Motion of a Particle w.r.t. a Rigid Body

So far we have always considered motion with respect to a fixed/inertial coordinate system. In many cases, especially in machines, it is useful to relate motions to a rotating coordinate system, specifically a coordinate system that is attached to a rotating rigid body.



P is a particle that is <u>not</u> fixed to the rigid body.

We are interested in describing the velocity and acceleration of the particle P.

$$\vec{r}_P = \vec{r}_A + \vec{r}_{P/A}$$

Here we assume that it is easiest to describe $\vec{r}_{P/A}$ in the x', y', z' coordinate system, i.e. the rotating system that is attached to the rigid body.

i.e. $\vec{r}_{P/A} = x'_P \vec{i}' + y'_P \vec{j}' + z'_P \vec{k}'$

or equivalently if a polar coordinate system is more appropriate

$$\vec{r}_{P/A} = r'_P \vec{e}'_r + z'_P \vec{k}'$$

We will focus on the primed Cartesian system, but our results will be completely general an will apply to the polar description as well.

$$\vec{v}_P = \frac{d\vec{r}_P}{dt} = \underbrace{\frac{d\vec{r}_A}{dt}}_{\vec{v}_A} + \frac{d\vec{r}_{P/A}}{dt}$$

Consider: $\frac{d\vec{r}_{P/A}}{dt} = \frac{d}{dt}\left(x'_P \vec{i}' + y'_P \vec{j}' + z'_P \vec{k}'\right)$

$$= \dot{x}'_P \vec{i}' + \dot{y}'_P \vec{j}' + \dot{z}'_P \vec{k}'$$

$$+ x'_P \frac{d\vec{i}'}{dt} + y'_P \frac{d\vec{j}'}{dt} + z'_P \frac{d\vec{k}'}{dt}$$

The first line is what is called "the velocity of the particle P with respect to the rigid body". This is the velocity that an observer sitting at point A and <u>rotating</u> with the rigid body would measure.

$$\vec{v}_{P/Body} = \dot{x}'_P \vec{i}' + \dot{y}'_P \vec{j}' + \dot{z}'_P \vec{k}'$$

Next, consider the terms $\frac{d\vec{i}'}{dt}$, $\frac{d\vec{j}'}{dt}$, $\frac{d\vec{k}'}{dt}$.

Recall that when we analyzed the motion of points <u>fixed</u> to the rigid body we encountered the quantity

$$\frac{d\vec{r}_{B/A}}{dt} = \vec{\omega} \times \vec{r}_{B/A}$$ where A & B are points fixed to the rigid body.

Hence, $\vec{r}_{B/A}$ was some vector that we can think of as being fixed to the rigid body.

Note that $\vec{i}'$, $\vec{j}'$ and $\vec{k}'$ are also vectors that are fixed to the rigid body, so

$$\frac{d\vec{i}'}{dt} = \vec{\omega} \times \vec{i}'$$

$$\frac{d\vec{j}'}{dt} = \vec{\omega} \times \vec{j}'$$

$$\frac{d\vec{k}'}{dt} = \vec{\omega} \times \vec{k}'$$

$\therefore$ The second line of our $\frac{d\vec{r}_{P/A}}{dt}$ equation can be rewritten as

$$x'_P \frac{d\vec{i}'}{dt} + y'_P \frac{d\vec{j}'}{dt} + z'_P \frac{d\vec{k}'}{dt} = x'_P\, \vec{\omega} \times \vec{i}' + y'_P\, \vec{\omega} \times \vec{j}' + z'_P\, \vec{\omega} \times \vec{k}'$$

then we can regroup terms as

$$x'_p \vec{\omega} \times \vec{i}' + y'_p \vec{\omega} \times \vec{j}' + z'_p \vec{\omega} \times \vec{k}' = \vec{\omega} \times \underbrace{\left(x'_p \vec{i}' + y'_p \vec{j}' + z'_p \vec{k}' \right)}_{\vec{r}_{P/A}}$$

$$= \vec{\omega} \times \vec{r}_{P/A}$$

$$\therefore \boxed{\vec{v}_P = \vec{v}_A + \vec{v}_{P/Body} + \vec{\omega} \times \vec{r}_{P/A}}$$

Recall our polar coordinate description of velocity, i.e.

$$\vec{v} = \dot{r}\, \vec{e}_r + r\dot{\theta}\, \vec{e}_\theta$$

$\vec{v}_{P/Body}$ is analogous to $\dot{r}$

$\vec{\omega} \times \vec{r}_{P/A}$ is analogous to $r\dot{\theta}$

<u>Now to acceleration</u>

$$\vec{a}_P = \frac{d\vec{v}_P}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d\vec{v}_{P/Body}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}_{P/A} + \vec{\omega} \times \frac{d\vec{r}_{P/A}}{dt}$$

Let's proceed term by term,

$$* \quad \frac{d\vec{v}_A}{dt} = \vec{a}_A$$

Recall $\vec{v}_{P/Body} = \dot{x}'_p \vec{i}' + \dot{y}'_p \vec{j}' + \dot{z}'_p \vec{k}'$

$$\therefore \frac{d\vec{v}_{P/Body}}{dt} = \ddot{x}'_P \vec{i}' + \ddot{y}'_P \vec{j}' + \ddot{z}'_P \vec{k}' + \dot{x}'_P \frac{d\vec{i}'}{dt} + \dot{y}'_P \frac{d\vec{j}'}{dt} + \dot{z}'_P \frac{d\vec{k}'}{dt}$$

$$\vec{a}_{P/Body} = \ddot{x}'_P \vec{i}' + \ddot{y}'_P \vec{j}' + \ddot{z}'_P \vec{k}'$$

This is the acceleration that an observer rotating with the rigid body would measure.

Then, $\dot{x}'_P \frac{d\vec{i}'}{dt} + \dot{y}'_P \frac{d\vec{j}'}{dt} + \dot{z}'_P \frac{d\vec{k}'}{dt} = \dot{x}'_P \vec{\omega} \times \vec{i}' + \dot{y}'_P \vec{\omega} \times \vec{j}' + \dot{z}'_P \vec{\omega} \times \vec{k}'$

$$= \vec{\omega} \times \vec{v}_{P/Body}$$

$$* \therefore \frac{d\vec{v}_{P/Body}}{dt} = \vec{a}_{P/Body} + \vec{\omega} \times \vec{v}_{P/Body}$$

$$* \frac{d\vec{\omega}}{dt} = \vec{\alpha} \rightarrow \frac{d\vec{\omega}}{dt} \times \vec{r}_{P/A} = \vec{\alpha} \times \vec{r}_{P/A}$$

$$* \frac{d\vec{r}_{P/A}}{dt} = \vec{v}_{P/Body} + \vec{\omega} \times \vec{r}_{P/A} \rightarrow \vec{\omega} \times \frac{d\vec{r}_{P/A}}{dt}$$

$$= \vec{\omega} \times \vec{v}_{P/Body} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$$

$$\therefore \boxed{\vec{a}_P = \vec{a}_A + \vec{a}_{P/Body} + 2\vec{\omega} \times \vec{v}_{P/Body} + \vec{\alpha} \times \vec{r}_{P/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})}$$

Compare this with our polar coordinate acceleration,

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$\vec{a}_{P/Body}$ is analogous to $\ddot{r}$

$2\vec{\omega} \times \vec{v}_{P/Body}$ is analogous to $2\dot{r}\dot{\theta}$

$-r\dot{\theta}^2$ is analogous to $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$

$\vec{\alpha} \times \vec{r}_{P/A}$ is analogous to $r\ddot{\theta}$

The book makes the following definitions. Call P' the point attached to the rigid body that resides at the same position as the particle P.

Then $\vec{v}_{P'/A} = \vec{\omega} \times \vec{r}_{P'/A} = \vec{\omega} \times \vec{r}_{P/A}$

$$\vec{a}_{P'/A} = \vec{\alpha} \times \vec{r}_{P'/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P'/A}) = \vec{\alpha} \times \vec{r}_{P/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})$$

Then $\vec{v}_P = \vec{v}_A + \vec{v}_{P'/A} + \vec{v}_{P/Body}$

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P'/A} + \vec{a}_{P/Body} + \vec{a}_c$$

$\vec{a}_c = 2\vec{\omega} \times \vec{v}_{P/Body} \equiv$ Coriolis acceleration