CUSTOM CROSS SECTION HYDROFORMING MODEL

by

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Significant insight into the hydroforming of relatively long tubes to a nearly uniform cross section can be developed by considering a section of the tube being compressed and pressurized under a state of generalized plane strain. A custom formulation and solution procedure developed for tackling such problems is outlined in this Appendix.

Kinematics

The problem consists of an initially circular tube of unit length, mean radius R and wall thickness t as shown in Fig. A1. The tube is loaded by internal pressure P and a uniform axial displacement Δ. The tube is enclosed in a rigid rectangular die with rounded corners. Because of symmetry, only one-fourth of the circumference is considered as shown in Fig.

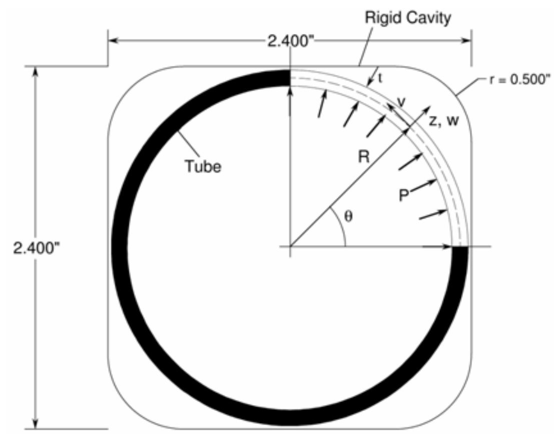

Fig. A1 Problem geometry

A cylindrical coordinate system with axial, circumferential and radial coordinates $\{x, R\theta, z\}$ is utilized. The displacement components of the mid-surface in the $(R\theta, z)$ directions are (v, w). The axial Lagrangian strain is given by

$$\eta_{xx} = \Delta + \tfrac{1}{2}\Delta^2. \tag{1}$$

The circumferential Lagrangian strain at any point in the tube in terms of the displacement components $\bar{v}$ and $\bar{w}$ of a general point on the cross section is given by

$$\bar{\eta}_{\theta\theta} = \left(\frac{r}{R}\right)^2 \left[\frac{\bar{v}' + \bar{w}}{r} + \frac{1}{2}\left(\frac{\bar{v}' + \bar{w}}{r}\right)^2 + \frac{1}{2}\left(\frac{\bar{v} - \bar{w}'}{r}\right)^2\right], \tag{2}$$

where $r = R + z$ and $(\bullet)' = d(\bullet)/d\theta$. If we assume that plane sections remain-plane,

$$\bar{v} = v - z^* \sin\beta \text{ and } \bar{w} = w - z + z^* \cos\beta. \tag{3}$$

The variable β is the rotation of the local normal and

$$\sin\beta = \frac{v - w'}{\lambda R},\ \cos\beta = \frac{1}{\lambda}\left(1 + \frac{v' + w}{R}\right), \tag{4}$$

where λ is the local stretch of the mid-surface and is given by

$$\lambda = \sqrt{1 + 2\varepsilon_\theta},\ \varepsilon_\theta = \eta_{\theta\theta}(z = 0). \tag{5}$$

Finally, z^* is the current distance of the point from the mid-surface given by

$$z^* = \int_0^z \sqrt{G_{zz}}\,dz \tag{6}$$

(G_{ii} and G^{ii} (no sum) are the covariant and contravariant components of the metric tensor).

Constitutive Behavior

The finite strain J_2 flow theory of plasticity is adopted. For plane stress ($\sigma_{zz} = 0$), the relationships between the Kirchoff stress and the Langrangian strain increments are given by

$$d\tau^{\alpha\beta} = \bar{L}^{\alpha\beta\gamma\delta} d\eta_{\gamma\delta} \quad (\alpha,\beta,\gamma,\delta = 1,2) \tag{7}$$

where

$$\bar{L}^{\alpha\beta\gamma\delta} = L^{\alpha\beta\gamma\delta} - \frac{L^{\alpha\beta 33} L^{33\gamma\delta}}{L^{3333}} \tag{8}$$

and

$$L^{ijkl} = \frac{E}{1+\nu}\left\{\left[\frac{1}{2}\left(G^{ik}G^{jl} + G^{il}G^{jk}\right) + \frac{\nu}{1-2\nu}G^{ij}G^{kl}\right] - \Gamma\left(\frac{f\bar{\tau}^{ij}\bar{\tau}^{kl}}{\frac{2}{3}\tau_e\left(\frac{1+\nu}{E} + f\tau_e\right)}\right)\right\}$$
$$-\frac{1}{2}\left(\tau^{ik}G^{jl} + \tau^{jk}G^{il} + \tau^{il}G^{jk} + \tau^{jl}G^{ik}\right). \tag{9}$$

Here, $\Gamma = 1$ for plastic loading and $\Gamma = 0$ for elastic deformation, $\bar{\tau}^{ij} = \tau^{ij} - \frac{1}{3}G^{ij}\tau^{kl}G_{kl}$ are the deviatoric stress components, $\tau_e = \left(\frac{3}{2}G_{ik}G_{jl}\bar{\tau}^{ij}\bar{\tau}^{kl}\right)^{1/2}$ is the equivalent stress and

$$f = \frac{3}{2\tau_e}\left\{\frac{\rho}{\rho_o}\left[1-(1-2\nu)\frac{\tau_e}{E}\right]\frac{1}{E_t} - \frac{1}{E}\right\}, \tag{10}$$

where ρ_o and ρ are the initial and current material densities. In the proposed work this formulation will be extended to included non-quadratic yield functions and/or to anisotropic ones.

Principle of Virtual Work

Since the axial strain is prescribed, the principle of virtual work (PVW) is given by

$$\int_0^{\pi/2}\int_{-t/2}^{t/2} \tau^{\theta\theta}\delta\eta_{\theta\theta}(R+z)\,dz\,d\theta = \delta W_c + \delta W_p \tag{11}$$

where the work done by the pressure is

$$\delta W_p = PR^2(1+\Delta)\int_0^{\pi/2}\left[\left(1+\frac{w}{R}+\frac{1}{2}\frac{v'}{R}\right)\frac{\delta w}{R} + \left(\frac{v}{R} - \frac{1}{2}\frac{w'}{R}\right)\frac{\delta v}{R} - \frac{1}{2}\frac{v}{R}\frac{\delta w'}{R} + \frac{w}{R}\frac{\delta v'}{R}\right]d\theta . \tag{12}$$

The contact between the tube and the die is modeled as “soft contact.” A set of M contact nodes are distributed around the circumference. As the gap (g) between the tube and the die closes, a reaction force builds up given by

$$F = \begin{cases} F_o\left(1+\dfrac{g}{g_o}\right)^b, & g > -g_o \\ 0, & g < -g_o \end{cases} \quad (13)$$

where F_o, g_o and b are parameters of the model. The contribution of the reactions at contact nodes to the PVW is

$$\delta W_c = R\sum_{m=1}^{M} F_m \delta g_m . \quad (14)$$

Solution

The mid-surface displacements are approximated using the following series expansions

$$v \cong R\sum_{n=1}^{N} b_n \sin 2n\theta,\; w \cong R\left[a_o + \sum_{n=1}^{N} a_n \cos 2n\theta\right]. \quad (15)$$

The discretization yields a system of $2N+1$ nonlinear algebraic equations for the unknown coefficients. At each loading step, increments in P and Δ are prescribed, and the equations are then solved using Newton's method. The integrations in (6) and (11) are carried out using numerical quadrature, while (12) can be easily integrated exactly. Parametric studies have indicated that N = 18, 36 Gauss integration points around the quarter-circle and seven points through the thickness are sufficient to obtain converged solutions.

The formulation is flexible enough so that it can be extended to other die cross sections relatively easily.