SYSTEM THEORY

1. Find the closest point(s) between the origin and the hyperboloid given by:

$$\frac{3}{2}x^2 - xz - y^2 + \frac{3}{2}z^2 = 4$$

2. For the equation given by:

$$2\dot{y}t^2 - 3yt - y^2 = 0,$$

where $\dot{y} = \frac{dy}{dt}$.

   a. Determine if the equation is homogeneous of degree zero (show why or why not).
   b. Find the solution for $y(t)$, when $y(1) = 2$.

3. Find the control history $u(t)$ that minimizes the performance index

$$J = \frac{1}{2}t_f^2 + \frac{1}{2}\int_{t_0}^{t_f} u^2 dt$$

subject to the differential constraints

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u$$

and the prescribed boundary conditions

$$t_0 = 0. \quad x_{1_0} = 0, \quad x_{2_0} = 0, \quad x_{1_f} = 1.$$

4. a. Apply the corner conditions to Prob. 3 to show that there cannot be a jump in the control.
   b. Apply the Weierstrass condition to Prob. 3 to show that the optimal solution is not a maximum.
   c. Derive the Legendre-Clebsch condition from the Weierstrass condition, and show that it is satisfied for Prob. 3.

5. a. Two scalar observations are taken of the scalar parameter x:

$$y_1 = x + \epsilon_1 \qquad y_2 = x + \epsilon_2$$

where ϵ_1 and ϵ_2 are zero mean random variables. It is known that the variance of ϵ_2 is twice the variance of ϵ_1, and that ϵ_1 and ϵ_2 are independent random variables. Determine the linear unbiased minimum variance estimate of x.

b. Calculate the variance of the estimate.

6. Assume you are given data sets r and s. Data set r has linearized data equations

$$y_r = H_r x_r + \epsilon_r$$

where y_r is $m_r \times 1$, H_r is $m_r \times n$, $x_r = x(t_r)$ is the $n \times 1$ deviation away from the nominal trajectory at t_r, and ϵ_r is $m_r \times 1$, where $m_r >$n. The weight matrix for data set r is W_r. Using data set r, calculate the weighted least squares estimate of x_r:

$$\hat{x}_r = (H_r^T W_r H_r)^{-1} H_r^T W_r y_r = P_r H_r^T W_r y_r$$

$$P_r \equiv (H_r^T W_r H_r)^{-1}$$

Data set s has linearized data equations

$$y_s = H_s x_s + \epsilon_s$$

where y_s is $m_s \times 1$, H_s is $m_s \times n$, $x_s = x(t_s)$ is the $n \times 1$ deviation away from the nominal trajectory at t_s, and ϵ_s is $m_s \times 1$. The weight matrix for data set s is W_s.

Derive the expression for the weighted least squares estimate of x at t_s using data sets r and s, where data set r is included as an apriori estimate.

You may use, without derivation, the properties of the state transition matrix. You may assume that you can obtain the value of any state transition matrix needed. You may assume that you know that the deviation $x(t)$ can be mapped to a different time using the state transition matrix as follows:

$$x(t_k) = \Phi(t_k, t_j) x(t_j)$$