Department of Aerospace Engineering and Engineering Mechanics

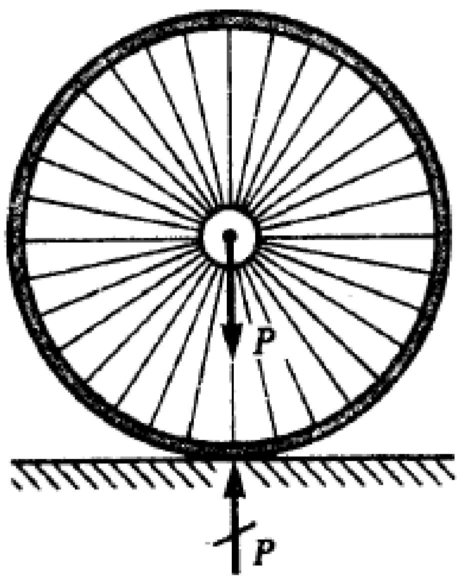
PhD Written Qualifying Examination
Friday, June 13, 2008
9:00 am – 12 noon

Answer all questions

THE UNIVERSITY OF TEXAS AT AUSTIN

1. The forces in the spokes of a bicycle wheel under a statically applied load may be determined approximately by considering the rim to be rigid and the spokes to be radial. The spokes are prestressed in tension so that they can carry a compressive load. Using Castigliano's first theorem, determine the downward displacement δ of the hub due to a force P if there are 32 equally spaced spokes having cross-sectional area A, length L, and modulus of elasticity E.

Note that $\sum_{n=1}^{7} \sin^2 \frac{n\pi}{16} = \frac{7}{2}$.



2. A torque tube consists of two concentric thin-walled tubes ($R >> t$) that are connected to rigid disks at the ends that ensure that they twist as a single unit. The outer tube has radius 3/2*R* and the inner one *R* while both have wall thickness *t*. The tubes have length *L* shear modulus *G* and the same yield stress. Furthermore the material is elastic-perfectly plastic with an axial yield stress of σ_o.

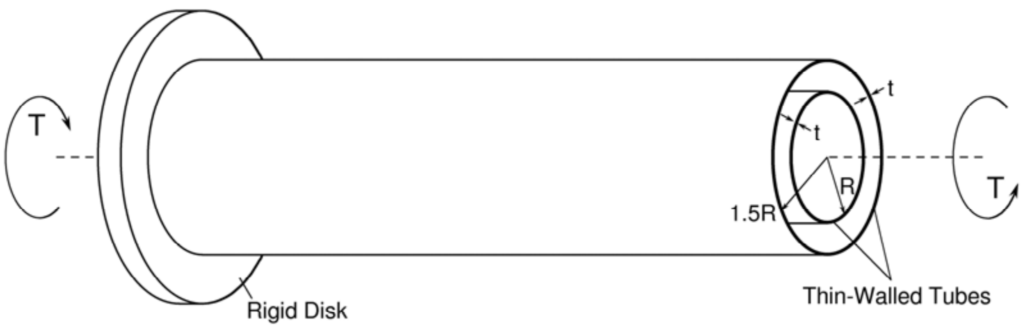
   (a) Calculate initial elastic the torsional rigidity of the composite tube and the torque at first yielding.

   (b) Calculate the torsional rigidity after one of the tubes has yielded and the maximum torque the composite tube can support.

   Guidelines:

   - Use the Tresca yield criterion.

   - Sketch the torque vs. the angle of twist per unit length (ϕ / L) and indicate your results on it.



3. A fiber pullout test is commonly used to evaluate the adhesion of fibers to a matrix. A long elastic circular fiber embedded in a matrix is pulled by a force F_o applied to a free end that is outside the matrix as shown in the figure. The embedded fiber with modulus E elongates and simultaneously transfers shear stress to the matrix. Follow the steps below to find how the force in the fiber decays with x assuming that the shear stress, $\tau(x)$, between the fiber and the matrix is related to the fiber axial displacement $u(x)$ through:

$\tau(x) = Cu(x)$, where C is a known constant.

(a) Write down the incremental equilibrium equation that relates the axial force $F(x)$ in the fiber to the shear stress $\tau(x)$.

(b) Now relate the axial displacement to the axial force and to the shear through part (a) to obtain the governing differential equation for $u(x)$.

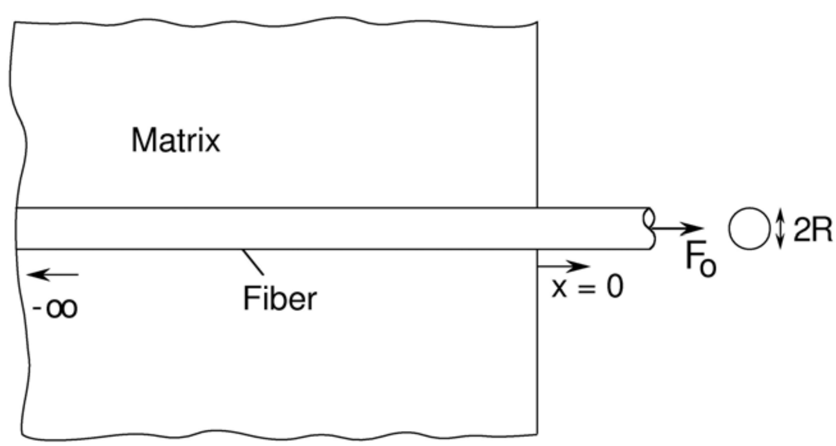
(c) Solve the differential equation with the appropriate boundary conditions ($x = -\infty$ and $x = 0$). Use the solution to find $F(x)$.

(d) BONUS: The shear strength of the glue cannot remain proportional to the displacement as given above; a more realistic representation is

$$\tau(x) = \begin{Bmatrix} Cu(x) & u < u_c \\ \tau_0 & u > u_c \end{Bmatrix}$$

What modifications are necessary when $u > u_c$?

Hint: Assume the fiber Poisson's ratio to be zero.



4. A *long* solid cylinder of radius a is rapidly heated around its surface, producing an instantaneous temperature distribution, $T = T_0 + T_1 \frac{r^2}{a^2}$, with r being the distance from its center. Assume that the solid remains linearly elastic with Young's modulus E and Poisson's ratio ν, and the coefficient of thermal expansion α is constant everywhere.

   a. Adopt the stress function ψ defined below and verify that the equilibrium equations are automatically satisfied.

   $$\sigma_r = \frac{\psi}{r}, \ \sigma_\theta = \frac{d\psi}{dr}$$

   b. From the strain-displacement relations, noting the symmetries of this problem, determine the only nontrivial compatibility equation among the strain components.

   c. Determine the governing equation for ψ and then solve this equation.

   d. Finally, determine the stress distribution in the cylinder.

5. The disk shown below has a density ρ and rotates at a constant angular speed ω. It has a variable thickness, $t(r)$ (note that this is a smooth variation).

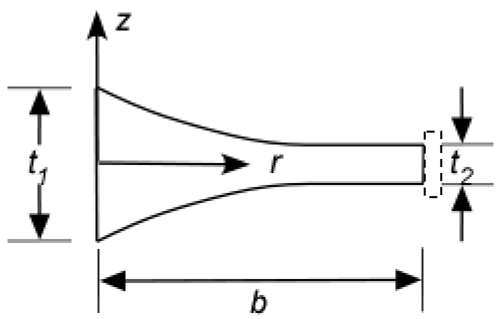
   a. Using a free body diagram of a representative element and considering the symmetry in the problem, show that the governing equilibrium equation is

$$\frac{d}{dr}\left(tr\sigma_r\right) - t\sigma_\theta + t\rho\omega^2 r^2 = 0$$

   b. Determine the variation in thickness that results in a constant equibiaxial stress.

   c. Show that the stress state is compatible.

   d. Does the stress state satisfy the traction at $r = b$? Discuss how you might address this with a rigid ring represented by the dashed rectangle?



FIELD EQNS IN CIRCULAR CYLINDRICAL COORDINATES

Equilibrium Equations

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_{r\theta}}{r\partial \theta} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + R = 0$$
$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{\theta}}{r\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + \Theta = 0$$
$$\frac{\partial \sigma_{zr}}{\partial r} + \frac{\partial \sigma_{\theta z}}{r\partial \theta} + \frac{\partial \sigma_{z}}{\partial z} + \frac{\sigma_{zr}}{r} + Z = 0$$

Strain-Displacement Relations

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \; \varepsilon_\theta = \frac{u_r}{r} + \frac{\partial u_\theta}{r\partial \theta}, \; \varepsilon_z = \frac{\partial u_z}{\partial z}$$
$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), \; \varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right), \; \varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right)$$

Stress-Strain Relations

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right] + \alpha \Delta T$$
$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu\left(\sigma_z + \sigma_r\right)\right] + \alpha \Delta T$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right] + \alpha \Delta T$$
$$\varepsilon_{r\theta} = \frac{\left(1+\nu\right)}{E}\sigma_{r\theta}$$
$$\varepsilon_{z\theta} = \frac{\left(1+\nu\right)}{E}\sigma_{z\theta}$$
$$\varepsilon_{zr} = \frac{\left(1+\nu\right)}{E}\sigma_{zr}$$