**Orbital Mechanics Written Qualifying Exam**
Department of Aerospace Engineering and Engineering Mechanics
The University of Texas at Austin
June 2007

Student Number: ______________________

Instructions: Answer all problems. There are five problems, each with a value of 20 points for a total exam value of 100 points. Problems with subparts (e.g., (a), (b), etc. will be equally weighted. Use a separate sheet for each problem. This is a closed book and notes examination. If you want the solution to a problem evaluated, you must write clearly and legibly.

1. A spacecraft is in an initial circular 225-km altitude orbit around the Earth.
   (a) Using a patched conic analysis, determine the magnitude of the delta-V maneuver needed to place the spacecraft on a hyperbolic departure trajectory that places the spacecraft on a heliocentric Hohmann transfer to Venus.

   (b) At Venus, the spacecraft performs a hyperbolic dark side flyby with a periapsis altitude of 11,125 km. What is the spacecraft post-flyby heliocentric velocity vector magnitude (in km/s) and orientation as specified by the flight path angle (in degrees)?

   (c) How long (in days) does it take for the spacecraft to return to Venus' heliocentric orbit radius?

For each of the sub-problems, outline the procedure along with all relevant equations needed. Then obtain a numerical result as requested. Assume that all planets are in coplanar circular orbits about the Sun.

2. Johannes Kepler published his empirical "laws of planetary motion" more than 50 years before Isaac Newton published his *Principia*. These laws [Plummer, 1918] are:
   1. The orbit of each planet is an ellipse with the Sun at one focus
   2. The heliocentric radius vector of each planet sweeps over equal area in equal time
   3. The square of the orbital period is proportional to the cube of the ellipse semimajor axis

Using Newtonian mechanics:
   (a) state the assumptions required for Law 1 to be valid
   (b) prove the Second Law of Kepler
   (c) prove the Third Law of Kepler

[Plummer, H.C., *An Introductory Treatise on Dynamical Astronomy*, Cambridge University Press, 1918.]

3. A spacecraft of infinitesimal mass is in equilibrium at the interior collinear libration point of the Earth-Moon system. Use the circular restricted problem of three bodies as the mathematical model of the system.

(a) Identify the forces that keep the spacecraft at the equilibrium point (i.e., why is it an equilibrium point?). Make an estimate of the magnitude of these forces, assuming the distance from the equilibrium point to the Moon is 15% (±1%) of the Earth-Moon distance and show that the conditions for equilibrium are met (to within 1% of the largest force). Express each force as a "force per unit mass" with units of acceleration in km/sec$^2$.

(b) If the Moon suddenly disappeared, determine the resulting orbit characteristics of the spacecraft and define appropriate reference frames for those characteristics. .

4. The linear stability properties of a periodic orbit that satisfies the equations of motion

$$\ddot{\mathbf{r}} = \mathbf{f}(\mathbf{r})$$

are determined by examining the eigenvalues of the Monodromy matrix $\Phi(T,0)$ associated with the periodic orbit. In the equations of motion, $\mathbf{r}$ is the position vector of a particle relative to an appropriate reference frame. The minimal period of the orbit is T.

(a) show that at least one of the eigenvalues of the Monodromy matrix is unity.

(b) explain or show why the orbit is linearly unstable if at least one of the eigenvalues of the Monodromy matrix has modulus greater than one.

5. A satellite orbits a spherical, nonrotating, planet (with constant density). The planet has an atmosphere, which produces a nongravitational force (atmospheric drag) on a low altitude satellite in a noncircular orbit. Let the atmospheric drag force per unit mass be expressed as $T_D$. Determine the effect of $T_D$ on the satellite's semimajor axis and inclination. State any assumptions.

Additional Information

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| Sun gravitational parameter | $1.327 \times 10^{11}$ km$^3$/s$^2$ |
| Earth gravitational parameter | $3.986 \times 10^{5}$ km$^3$/s$^2$ |
| Moon gravitational parameter | $4.902 \times 10^{3}$ km$^3$/s$^2$ |
| Venus gravitational parameter | $3.248 \times 10^{5}$ km$^3$/s$^2$ |
| Earth heliocentric circular orbit radius | $1.496 \times 10^{8}$ km |
| Venus heliocentric circular orbit radius | $1.082 \times 10^{8}$ km |
| Moon geocentric circular orbit radius | $3.844 \times 10^{5}$ km |
| Earth radius | 6378 km |
| Venus radius | 6053 km |
| Moon radius | 1738 km |

$\sin(\delta/2) = 1/e$ , $e = 1 + \mu^{-1} r_p V_\infty^2$ , $\tan(\nu/2) = \sqrt{(1+e)/(1-e)}\,\tan(E/2)$

where $\delta$ is the angle between the hyperbolic asymptotes or the "turn angle", $\nu$ is the true anomaly and E is the eccentric anomaly.