Orbital Mechanics Written Qualifying Exam

Department of Aerospace Engineering and Engineering Mechanics

The University of Texas at Austin

June 2005

Name____

Student Number:____________

Instructions. Answer all problems. There are six problems, each with a value of 20 points for a total exam value of 120 points. Use a separate sheet for each problem. This is a closed book and notes examination.

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1. A spacecraft in a 300-km by 2000-km altitude Earth orbit is to rendezvous with a second spacecraft in a 600-km by 1000-km altitude orbit. The spacecraft orbit in the same orbit plane and have the same periapsis direction. What is the true anomaly of the second spacecraft when the first spacecraft is at periapsis to assure that an intercept will occur before the first spacecraft reaches apoapsis?

2. Consider a heliocentric spacecraft mission where the orbits of the planets are coplanar and circular.

   (a) Construct a Hohmann transfer from Earth to Jupiter

   (b) Design a flyby of Jupiter such that the heliocentric post-encounter velocity is maximized. Assume that Jupiter has no atmosphere. What is the Jupiter centered eccentricity and the turning angle?

   (c) Determine if it is necessary to apply a velocity impulse at periapsis of the Jupiter flyby to assure the spacecraft will escape the solar system. If there is a maneuver, what is its value?

   In your answers please provide sketches of the heliocentric Hohmann transfer and the Jupiter flyby diagram and geometry. Label all appropriate quantities.

3. The equations of motion for a system of n ($n \geq 2$) point masses that are under their mutual gravitation is

$$\frac{d^2\mathbf{r}_i}{dt^2} = G\sum_{j=1}^{n}\frac{m_j}{r_{ij}^3}(\mathbf{r}_j - \mathbf{r}_i) \qquad i = 1, ..., n \qquad j \neq i$$

Where it is assumed that the position vectors are referenced to an inertial reference frame and $\mathbf{r}_i$ is the position of body i and $\mathbf{r}_j$ is the position of body j. G is the universal constant of gravitation and m_i is the mass of particle i. The distance between them is $r_{ij} = \sqrt{(\mathbf{r}_j - \mathbf{r}_i)^\top(\mathbf{r}_j - \mathbf{r}_i)}$. Assume that there are no external forces acting on the system.

(a) If we define a scalar potential function to be

$$U = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{2}\frac{Gm_im_j}{r_{ij}}$$

show that the force on particle i can be written as the gradient of this potential with respect to the absolution position vector of particle i

$$\mathbf{f}_i = \left(\frac{\partial U}{\partial \mathbf{r}_i}\right)^\top$$

(b) If the total kinetic energy of the system is

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2$$

where v_i is the absolute velocity of particle i, show that the total energy of the system

$$E = T - U$$

is an integral of the system.

4. The planar form of the circular restricted three body problem in rotating coordinates is described by the following equations of motion

$$\begin{aligned} \ddot{x} &= +2\dot{y} + \frac{\partial U}{\partial x} \\ \ddot{y} &= -2\dot{x} + \frac{\partial U}{\partial y} \end{aligned}$$

where

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

μ is the dimensionless mass ratio parameter

$$\mu = \frac{m_2}{m_1 + m_2} \qquad (m_2 < m_1)$$

and r_1 and r_2 are the distances between the particle and the mass particles 1 and 2, respectively. m_1 with dimensionless mass $1 - \mu$ is located at $x = -\mu, y = 0$. m_2 with dimensionless mass μ is located at $x = 1 - \mu$, $y = 0$.

Derive any integrals the system may possess and discuss its(their) utility and significance.

5. An Earth satellite with an ion propulsion system continuously thrusts in a direction perpendicular to both the position and velocity vectors. Derive an expression for the time rate of change of p, the semi-latus parameter of the orbit.

6. The Apollo Lunar Module was launched from the surface of the moon starting from rest. The mass of the vehicle at launch is m_0. m_0 is composed of a dry mass and propellant mass. The burnt propellant is expelled from the rocket nozzle with an exhaust velocity v_e. Derive the equations of motion, assuming the vehicle rises vertically and the acceleration due to gravity is constant. Identify the term(s) in the equation of motion known as 'thrust'.

Constants and other useful information

$$\mu_{earth} = 3.986 \times 10^5 \text{ km}^3/\text{s}^2$$
$$\mu_{jupiter} = 1.267 \times 10^8 \text{ km}^3/\text{s}^2$$
$$\mu_{sun} = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2$$
$$a_{earth} = 1.496 \times 10^8 \text{ km}$$
$$a_{jupiter} = 7.784 \times 10^8 \text{ km}$$
$$r_{earth} = 6378 \text{ km}$$
$$r_{jupiter} = 71497 \text{ km}$$

If E is the eccentric anomaly and ν is the true anomaly, then

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2}$$

If v is the magnitude of the inertial referenced velocity, r is the distance from the gravitating body, and μ is the gravitational parameter of the gravitating body then

$$Energy = \frac{v^2}{2} - \frac{\mu}{r}$$

If p is the semi-latus parameter, e the eccentricity then

$$r = \frac{p}{1 + e\cos\nu}$$

If a is the semi-major axis then

$$p = a(1-e^2)$$

If h is the magnitude of the angular momentum then

$$h = \sqrt{p\mu}$$

If the orbit is parabolic/hyperbolic and δ is the turning angle, then

$$\delta = 2\sin^{-1}\frac{1}{e}$$

If the orbit is parabolic/hyperbolic, r_p is the periapsis radius, and v_∞ is the magnitude of the hyperbolic excess velocity then

$$e = 1 + \frac{r_p v_\infty^2}{\mu}$$