ASE/EM Math Qualifying Examination
Date: 2007.

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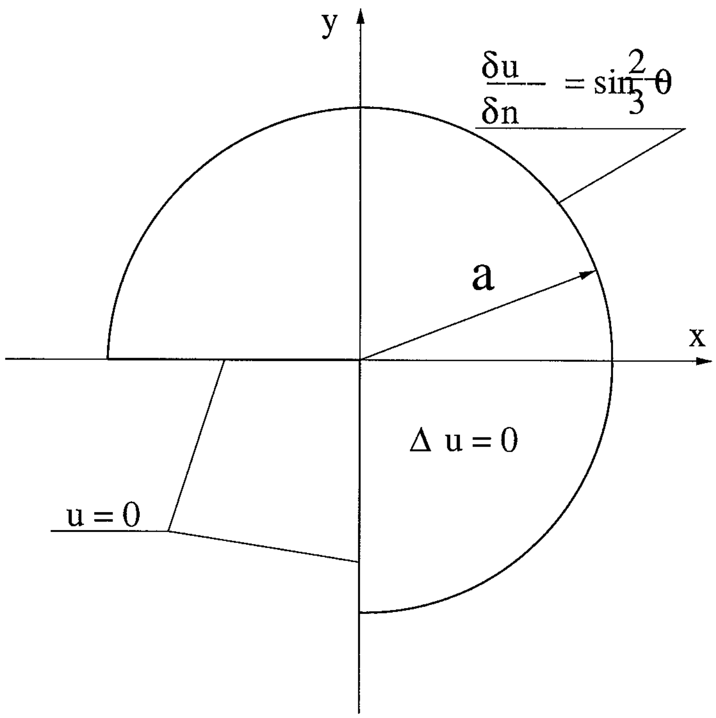
1. Find the general solution to the following system of ODE's.

$$\begin{cases} 2\dot{x} + \dot{y} - 2x - 3y &= 4t \\ \dot{x} + \dot{y} - x - 2y &= 3t \end{cases}$$

where $\dot{x} = \frac{dx}{dt}$.

2. Use separation of variables to find a bounded solution of the boundary value problem depicted in the figure below.



3. Find the closest point(s) between the origin and the hyperboloid given by:

$$\frac{3}{2}x^2 - xz - y^2 + \frac{3}{2}z^2 = 4$$

4. Consider the equation given by:

$$2t^2\frac{dy}{dt} - 3ty - y^2 = 0.$$

(a) Determine if the equation is homogeneous-of-degree-zero (show why or why not).

(b) Find the solution for $y(t)$, when $y(1) = 2$.

5. Consider a hollow sphere $a^2 \leq x^2 + y^2 + z^2 \leq b^2$. Suppose the mass density function $\sigma(x, y, z)$ over the sphere is constant.

(a) Show that the total mass of the sphere is

$$M = \frac{4}{3}\pi(b^3 - a^3)\sigma.$$

(b) The moment of inertia about the x axis for a volume V is defined as

$$I_x = \int\int\int_V (y^2 + z^2)\sigma dV.$$

Show that for the hollow sphere

$$I_x = \frac{2}{5}M\frac{b^5 - a^5}{b^3 - a^3}.$$

(c) In the limit as $a \to b$, the hollow sphere becomes a thin shell. Compute I_x for a thin shell of radius b.

6. Suppose C is a simply closed contour oriented counterclockwise and bounding the region R. Suppose also that u and v are continuous functions with continuous first partial derivatives in R. Then Green's Theorem states that

$$\int_C udx - vdy = -\int\int_R \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] dxdy.$$

(a) Verify Green's Theorem for $u = x^2 - y^2$, $v = -2xy$ and R the triangle with vertices (0,0), (2,0) and (2,2).

(b) Suppose f is the complex function $f(z) = u + iv$. Use Green's Theorem to prove the Cauchy-Gorsat Theorem.