Fluid Mechanics PhD Qualifying Examination
June 2008

READ THESE INSTRUCTIONS.

Answer all **six** questions. Please write on one side of the paper only and put your Code Number and appropriate question number on every sheet. **Begin each question on a separate sheet of paper.** To obtain complete credit, your work must be neat and your complete procedure shown. Draw neat sketches and list your assumptions. Ask for clarification if the meaning of a question is unclear to you.

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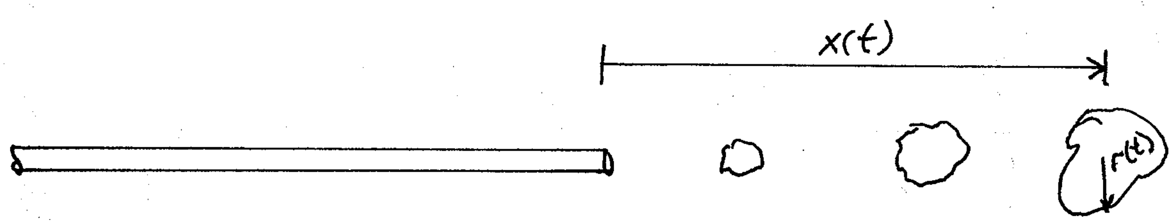
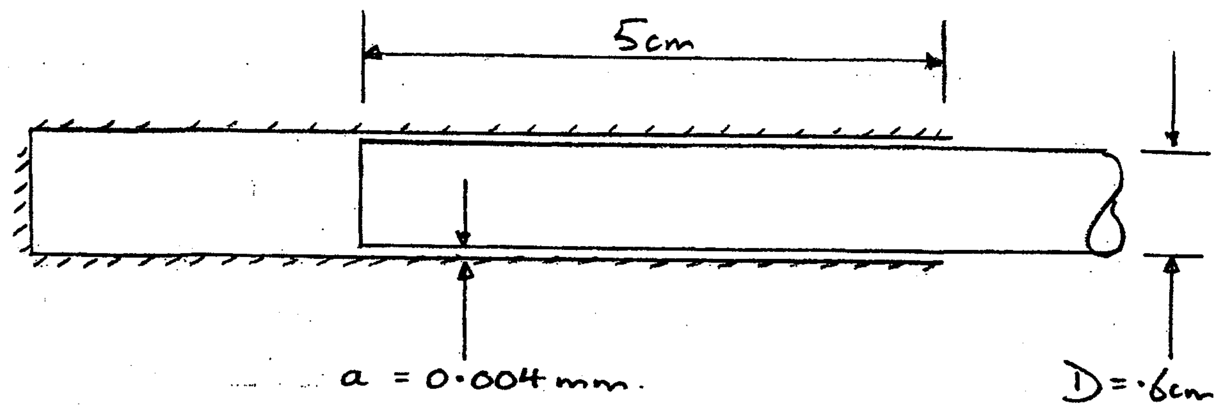
1. A puff of air, density ρ, marked by smoke, is suddenly ejected from the end of a pipe into an infinite domain. (This is like a small turbulent vortex ring.) As the puff moves away from the end of the pipe, it grows in volume by turbulent entrainment of the surrounding air, yet it remains roughly spherical in shape. A movie is taken of the situation to find the distance, x(t), the puff has moved and its average radius, r(t). We can measure the instantaneous horizontal puff velocity as dx/dt. The ejection of the puff imparts a total horizontal momentum M that remains constant over time as the puff mixes with the surrounding air.
   (a) Argue why M should be constant.
   (b) What is a simple expression for M in terms of the given measurable quantities?

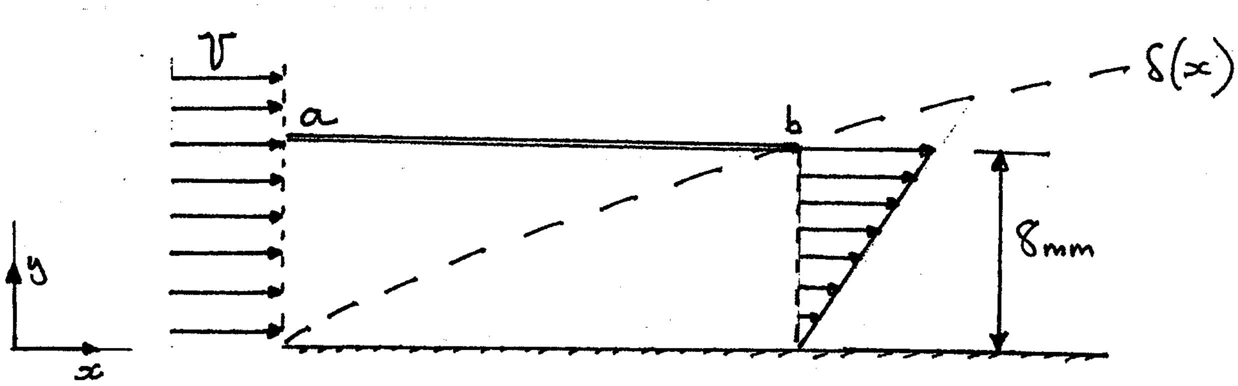
   The evolution of the puff is an unsteady turbulent flow in which quantities x and r depend on time and the parameters M and ρ but not upon viscosity μ. Using dimensional analysis, answer the following:
   (c) Derive expressions for the dependence of x and r upon t, ρ, and M.
   (d) Assuming that the kinetic energy per unit mass in the puff is proportional to the puff speed squared, show how the kinetic energy of the puff varies as a function of time, t.
   (e) The turbulent kinetic energy dissipation rate $\varepsilon \cong -(d/dt)(dx/dt)^2$. Determine how ε depends on t, ρ, and M.



3. A high pressure in a system is created by a small piston-cylinder assembly employing oil. The piston diameter is 0.6cm and extends 5cm into the cylinder, as shown below. The radial clearance between the piston and the cylinder is 0.004mm. Assuming: (i) elastic deformations of the piston and the cylinder can be ignored and (ii) that the oil being used has a density of 880kg/m^3 and coefficient of viscosity, μ, of 0.004Pa.s, calculate the leakage rate of oil around the piston when the pressure in the cylinder is 7000 atm..



4. Air flows over a thin flat plate that is 1m long and 0.3m wide. The air has a density of 1.21kg/m^3. The flow is uniform at the leading edge of the plate. Assuming that (i) the velocity profile in the boundary layer is linear, (ii) the free stream velocity is 2.7m/sec, (iii) the flow is two-dimensional; compute the mass flow rate across surface *ab* and determine the force required to hold the plate stationary.



ADDENDUM

The continuity and Navier-Stokes Equations in cylindrical coordinates (r, θ, z) for incompressible flow with constant properties are

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right)$$

$$= F_r - \frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right)$$

$$= F_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right)$$

$$= F_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$