FLUIDS
Ph.D. Qualifying Examination
June 2007

INSTRUCTIONS TO STUDENTS:

Answer all questions. Please write on one side of the paper only and put your code number and appropriate question number on every sheet. Begin each question on a separate sheet of paper. To obtain complete credit, your work must be neat and your complete procedure shown. Draw neat sketches and list your assumptions. Ask for a clarification is the meaning of a question is unclear to you. Note: there are six problems this year and problems 1 and 2 suggest a guide for the length of your response.

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1. a) Define the material derivative for transport of a scalar field ϕ in an eulerian frame in 3D. (1 line) Use this in an integral balance equation for transport of species or heat and derive the model transient convection-diffusion transport equation, explaining your mathematical reasoning(6 lines).

   (b) Consider the one-dimensional steady-state form of the model convection-diffusion equation,

   $$u\phi' - k\phi'' = 0$$

   with constant coefficients u, k and construct a central difference approximation to the equation, giving the explicit form of the leading truncation error.(7 lines) Apply a monotonicity (slope) argument to the difference equation to deduce the corresponding cell Reynolds (Peclet) number condition.(4 lines) Why is this inequality relevant to CFD?(1 line)

2. Assume a nonlinear reaction term $r \exp(\phi)$, parameter r, is added to the right hand side of the steady transport equation above in Problem1(b). The finite difference approximation now has a corresponding nonlinear term, so we now have a parameterised nonlinear algebraic system. Assume you are interested in numerical simulation for a range of increasing reaction rate values r beginning at $r=0$:

   (i) Derive the Newton iteration formula for a *general* nonlinear algebraic *system* and write out the contribution of the reaction term alone to the Newton Jacobian matrix for the discretized reaction diffusion system. (4 lines). Newton's method is said to converge "quadratically": What does this mean mathematically and when does it happen? (2 lines) What is meant by "domain of attraction" and what complications do you anticipate for the domain of attraction of Newton iteration as the reaction parameter value increases(2 lines)?

   (ii) If a norm of the solution is graphed against parameter r then one can associate solution branches having critical points such as bifurcation and turning points at certain parameter values. What do these parameter values correspond to in the case of a *linear* reaction $r\phi$. (1 line) When convection is absent in our model problem, a turning point occurs for the simplified *nonlinear* diffusion-reaction problem near $r=4$. Draw a rough sketch showing the branch behavior through this turning point. What numerical complications do you anticipate for reaction parameter values near the turning point and why?(1 line) Indicate a strategy for circumventing some of the difficulties at the turning point and explain how it addresses the problem for Newton's method at the turning point (3 lines).

3. For a non-Newtonian fluid the relationship between shear stress and strain rate can be expressed as a power law

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n \qquad \text{.................. (1)}$$

where the exponent n is called the flow behavior index and k is the consistency index. In the case of Newtonian fluid n=1 and k=μ so equation (1) simply becomes

$$\tau_{yx} = \mu\left(\frac{du}{dy}\right) \qquad \text{....................(2)}$$

Show that for such a power law fluid the velocity profile for fully developed flow between horizontal parallel plates separated by distance 2h is given by

$$U = \left(\frac{h}{k}\frac{\Delta P}{L}\right)^{\frac{1}{n}} \frac{nh}{n+1}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

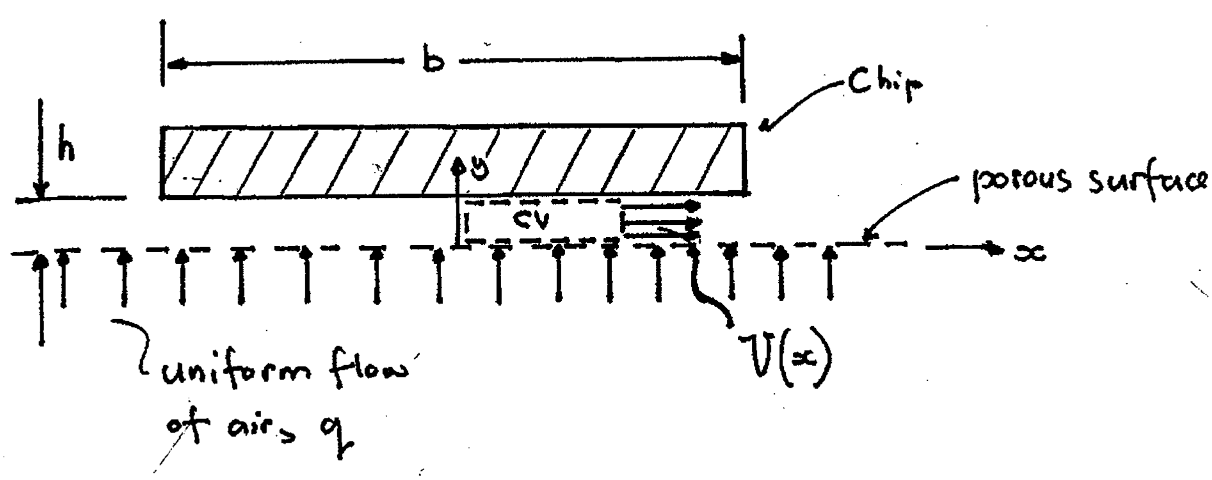
where Δ P is the pressure drop over distance L, and y is measured from the channel centerline.

4. A rectangular integrated circuit "chip" floats on a thin layer of air, h=0.5 mm thick, above a porous surface, as shown. The chip length L is very long in the direction into the paper, z. There is no flow in the z direction. You can assume that flow in the x direction in the gap under the chip is uniform, that the flow is incompressible and frictional effects can be neglected

   a. Show that $U(x) = q\frac{x}{h}$ in the gap where q is the volume flow rate per unit area through the porous surface (i.e., q has units of $m^3/s/m^2$)

   b. Obtain a general expression for fluid particle acceleration in the gap.

   c. Calculate the magnitude of the maximum acceleration of a fluid particle in the gap if $q = 0.06\ m^3/s/m^2$



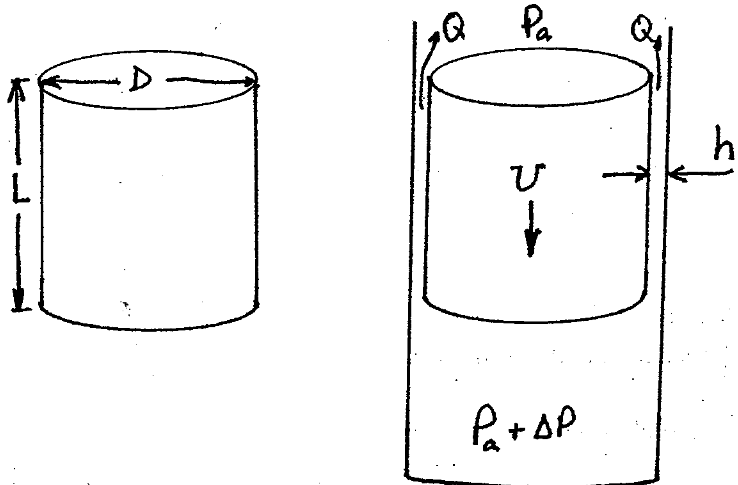
5. A solid metal cylinder of diameter D, length L and density ρ_m is placed inside of a hollow circular cylinder of slightly larger inside diameter, $D+2h$ where $h<<D$. That is, the gap between the cylinders is very small. When released, the solid cylinder is observed to fall slowly at a constant speed U. We ultimately want to calculate U, assuming that the airflow through the narrow gap of thickness h between the two cylinders is steady, laminar and incompressible. Denote the air density by ρ_{air} and viscosity by μ.

    a. Derive or simply write down an expression for the volumetric flow rate Q of air through the gap between the two cylinders in terms of U and the other geometric parameters.

    b. If the pressure above the solid cylinder is atmospheric at P_a and that below is $P_a+\Delta P$, derive an expression for ΔP in terms of μ, L, D, U and h.

    c. Derive an expression for the vertical shear stress, τ, on the outside lateral surface of the solid metal cylinder in terms of the quantities μ, L, D, U and h.

    d. Using a force balance on the metal cylinder, derive an expression for the speed of falling (U) in terms of gravity g and ρ_m, h, μ and D.



6. In the blow drying of a thin liquid film, the liquid evaporates and the vapor is swept away by the air moving past the surface of the film as seen in the figure. We are interested in the drying time, t, that depends on the film length, L, film thickness, δ, liquid vapor pressure, P_v, air speed U, the air viscosity, μ, and the air density, ρ.

   a. Derive (or you may simply write down) a set of dimensionless variables relating the drying time, t, to the other variables L, ρ, δ, P_v and U.

   b. We would like to design a laboratory experiment to determine the drying time of a soccer field covered with a 1cm thick layer of water under a 2m/s wind. The vapor pressure of water is 2000Pa, the length of the field is 100m. In the proposed experiment the air density and viscosity are the same as that for the soccer field but the model field will be only L_{mod} = 20m long. Calculate the values U_{mod}, δ_{mod} and $P_{v,mod}$ in order to ensure dynamic similarity with the full-scale flow.

   c. If the drying time measured in the laboratory model of part c) is t_m = 10min, calculate the time to dry the soccer field.

   d. Briefly explain why the liquid density and viscosity should not also have been included in this list of important variables. Then, within three or four sentences, comment on the practicality of actually doing this experiment. What problems might arise?