

**EM 397 (Unique Number 13595) or  
CAM 383 (Unique Number 65680) or  
PGE 383 36 (Unique Number 19265)**

**Advanced Numerical Algorithms**

**Time: MWF, 1-2 pm**

**Location: CBA 4.336**

**Instructor: Mary Fanett Wheeler**

**Office: ACES 5.324**

**Office Hours: TuTh 1:30 - 3:00pm or by appointment  
(mfw@ices.utexas.edu, or call connie at 475-8625)**

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**Course Objectives**

- Develop basic mathematical formulations and numerical algorithms for modeling flow, transport, biogeochemistry and geomechanics in porous media.
  - Mathematical formulations covered will include elliptic, parabolic transported dominant with reactions, elasticity and elastic wave propagation.
  - Numerical discretization algorithms covered will include finite differences and finite elements, e.g. mixed methods (mimetic, MPFA, general element), discontinuous Galerkin, and conforming Galerkin with projections for conservation.
- Develop an understanding of convergence and error estimators for the above discretization schemes. This will include both a priori and a posteriori error estimates.
- Discuss basic iterative coupling schemes for multiphysics couplings.
- Discuss schemes for treating non-matching grids and multiscale problems, both mortar and mortar-free.
- Even though computational issues emphasize porous media applications, algorithms discussed are used in biomedical, mechanical, environmental and aerospace engineering applications.

**References**

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- Saad, Y. , *Iterative Methods for Sparse Linear Systems*, PWS, Boston (1996).
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- Thompson, J. and Martin, C. , *Numerical Grid Generation*, North Holland, NY (1985).
- Zienkiewicz, O., Chan, A. , Pastor, M. , Schrefler, B. and Shiomi,T. , *Computational Geomechanics*, Wiley, NY (2001).

## Course Outline

- Mixed Finite Element Methods for Elliptic Problems (A Priori and A Posteriori Results)
  - Standard Mixed Finite Element Formulation with Mortar Interfaces
  - Relation to Mimetic and MPFA
  - Extensions of MFE to General Elements
  - A Posteriori Results (Ainsworth and Vohralik et. al)
- Discontinuous Galerkin for Elliptic Problems
  - Standard A Priori Analysis
  - A Posteriori Results
  - Coupling with MFE with Mortars or by Hybridization
- Extensions to Parabolic Equations
- Iterative Coupling and Time Stepping
- Extensions to Elasticity and Poroelasticity
- Multiphysic Couplings

## Grading

- Homework. (30 percent).
- Projects. Assignments Based on Interests and Backgrounds. Two Person Teams May Work Together.
- Midterm. Progress Report on Projects (15 percent).
- Final. (30 percent).
- Final Written Report on Projects and Oral Presentation. (25 percent).

## Prerequisite: Graduate Standing

**Evaluation Plan: Evaluation will be conducted at the end of the semester.**