

EM394H/CAM394H ADVANCED THEORY OF FINITE ELEMENT METHODS

Spring 08, # 13575/ # 65775 , MWF 10:00-11:00, WRW 413

The class covers fundamentals of convergence analysis for Finite Element Method and linear boundary-value problems. We will emphasize discretizations building in two exact sequences: the classical grad-curl-div sequence, and the less known elasticity complex.

We shall begin with a short review of abstract variational problems and corresponding convergence results for the Galerkin method: Lax-Milgram Thm., Babuška's Thm., Babuška - Brezzi theory for mixed problems, indefinite problems and asymptotic convergence.

We will review then a number of classical examples related to time-harmonic acoustics, elasticity and Maxwell equations.

Half of the semester will be devoted to a study of relevant *energy spaces*: the classical Sobolev spaces, spaces $H(\text{curl})$, $H(\text{div})$ and the energy spaces corresponding to the elasticity complex. This part of the class will review the most recent research results in the field.

The second part of the lectures will be devoted to the convergence analysis of the Finite Element Method including results on h , p and hp interpolation.

CLASS OUTLINE SCHEDULE:

Jan 14 - Jan 18 Examples of variational (weak) formulations: linear acoustics, Maxwell equations, elastodynamics, coupled problems.

Jan 23 Jan 25 Abstract Variational Boundary-Value problems, Generalized Lax - Milgram Thm, Galerkin method, Cea's Lemma, Babuška's Thm.

Jan 28 - Feb 1 Babuška-Brezzi theory, Compactly perturbed problems and asymptotic stability.

Jan 30 - Feb 3 Distributions and Sobolev spaces.

Feb 4 - Feb 8 Sobolev spaces: alternative definitions, fractional Sobolev spaces, Density and Imbedding Thms.

Feb 11 - Feb 15 Trace Theorem, boundary spaces, generalized Green's formula.

Feb 18 - Feb 22 $H(\text{curl})$ and $H(\text{div})$ spaces, and the corresponding boundary spaces.

Feb 25 - Feb 29 $H(\text{curl})$ and $H(\text{div})$ spaces - cont.

Mar 3 - Mar 7 Nedelec and Raviart-Thomas elements, discrete exact sequences.

Mar 17 - Mar 21 Elasticity complex.

Mar 24 - Mar 28 Classical interpolation theory, Bramble-Hilbert Lemma.

Mar 10 - Mar 20 Aubin-Nitsche trick, convergence in weaker norms. Strang's lemmas, analysis of quadrature error.

Mar 31 - Apr 4 Convergence analysis for elliptic eigenvalue problems, elements of spectral theory.

Apr 7 - Apr 11 Asymptotic stability and convergence. Wave propagation problems, pollution (dispersion) errors.

Apr 14 - Apr 18 h -adaptivity and how it restores optimal rates of convergence, Babuška-Kellog-Pitkaranta theory, main ideas of a-posteriori error estimation.

Apr 21 - Apr 25 Mixed formulations, relation between inf-sup condition and exact polynomial sequences, Maxwell equations.

Apr 28 - May 2 The p and hp -methods. Projection-based interpolation and convergence analysis.

The class will be conducted in a seminar style. No exams (including final) will be given. Instead, problems with varying difficulty, ranging from "hard theory", through practice exercises, to assignments involving numerical experiments, will be assigned in the class on a continuous basis. Each problem will be worth a number of points (5-20). The final grade will be determined by the number of collected points.

Final score range	grade
100 and above	A+
80 - 100	A
60 - 80	B
40 - 60	C
20 - 40	D
00 - 20	F

Discussion session; one hour, once a week, to be determined.

Instructor: Dr. Leszek Demkowicz, ACES 6.326, Office hours: immediately after the class.