

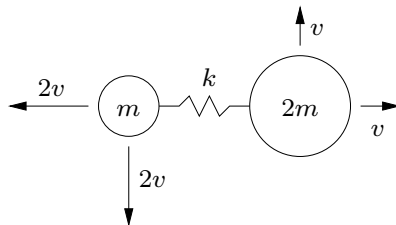
Dynamics Qualifying Exam — 2008

WORK ALL 5 PROBLEMS.

Problem 1

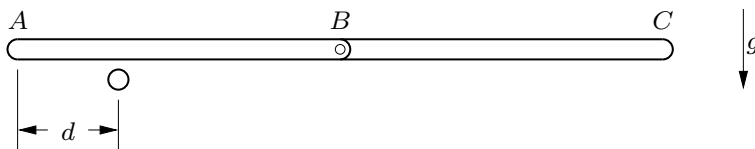
The two masses shown slide without friction on the horizontal plane, and are connected with a spring of unstretched length L and spring constant k . The masses have the initial velocity components shown, and the spring is initially unstretched.

- What are all of the dynamic quantities that are conserved in the motion of this system? For the initial conditions given, what are each of these quantities equal to?
- Derive an equation that can be solved for the maximum stretch of the spring with the given initial conditions.
- If the equation obtained for (b) is linear or quadratic, solve it for the maximum spring elongation. If it is of cubic or higher order, discuss what types of solutions you expect to find and the physical meanings of the solutions.



Problem 2

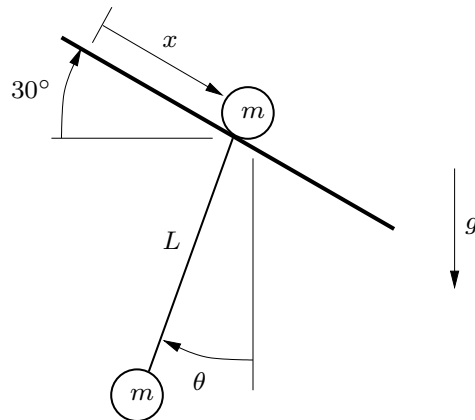
Two identical rods, each of mass m and length L , are pinned together and are falling with velocity v when the left rod AB strikes a rigid stationary obstruction as shown. If the coefficient of restitution is e and the angular velocity of the right rod BC is equal to zero immediately after the collision, what is the distance d from the left end of rod AB to the obstruction? What are the angular velocity and the velocity of the mass center of rod AB immediately after impact?



Problem 3

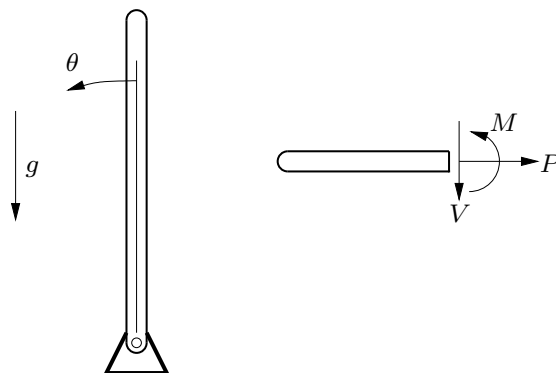
Two identical masses are connected by a string of length L , and the upper one slides without friction down a slope inclined at 30° . Let x be the distance the upper mass travels, and let θ be the angle between the string and the vertical.

- (a) If θ is constant in the motion of this system, what is its value?
- (b) What is the natural frequency for small oscillations of this system about the motion of part (a)?



Problem 4

A rod of mass m and length L is pinned at one end and is released from an upright position as shown, to rotate through angle θ . Find the axial force P , shear force V and bending moment M at the rod's center, as functions of θ .



Problem 5

One edge of a thin square rigid plate, of mass m and dimensions a , is welded to a rigid rod that can rotate freely about a horizontal axis. The rod is supported by a rigid frame which can rotate freely about the vertical axis. One edge of the plate coincides with the vertical axis of rotation when the plate is vertical. The mass moment of inertia of the frame about the vertical axis of rotation is I_F . Let θ be the frame's angle of rotation about the vertical, and ϕ be the plate's angle of rotation about the horizontal, measured from the downward vertical position. Suggested axes: at the instant pictured (when θ and ϕ are both zero), the x axis is aligned with the horizontal rod, the y axis is vertical, and the z axis is perpendicular to the plate and the frame.

- Obtain equations of motion governing θ and ϕ when the frame and the plate rotate freely.
- If the frame is rotated with constant angular velocity $\dot{\theta} = \Omega$, find the largest value of Ω such that the position $\phi = 0$ is stable.

