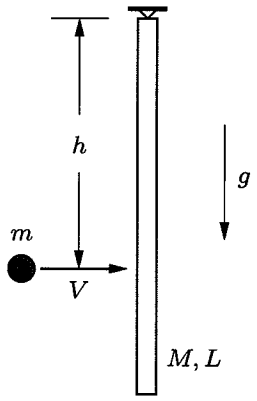
Dynamics Qualifying Exam — 2007

WORK ALL 5 PROBLEMS.

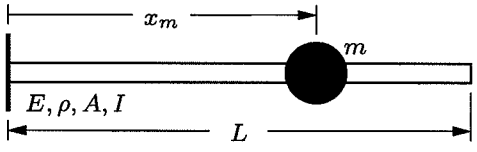
Problem 1

A rod of mass M and length L is suspended from a frictionless pin joint and is struck by a projectile of mass m that has horizontal velocity V. Find the distance h from the pin joint to the point of impact such that the projectile's horizontal velocity immediately after impact is zero. The coefficient of restitution is e.



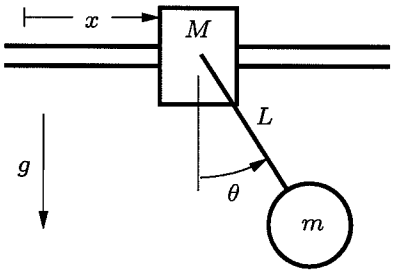
Problem 2

A particle mass m is to be attached to a uniform cantilever beam of length L, Young's modulus E, mass density ρ, cross-sectional area A and area moment of inertia I. Use an appropriate admissible function $\psi(x)$ to approximate the attachment location x_m such that the lowest natural frequency of the system is the same as if the added mass were distributed uniformly along the length of the beam (increasing the mass per unit length ρA by m/L).



Problem 3

A pendulum of mass m and length L is suspended from a slider of mass M which moves horizontally without friction. If the slider and pendulum are initially at rest, with $x = 0$ and $\theta = 0$, when an impulse is applied to the slider to give it an initial velocity V, find the minimum initial slider velocity V such that the pendulum will rotate to a vertical position above the slider.



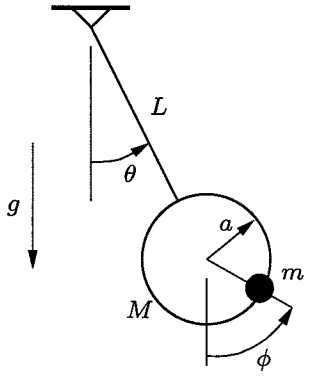
Problem 4

A system consists of a rigid hoop of mass M and radius a on which a small bead (point mass) of mass m can slide without friction. An arm of length L is rigidly welded to the hoop and swings from a pin joint through angle θ without friction. All motion takes place in the plane of the hoop, which is vertical.

(a) Using the coordinates shown, derive equations of motion for the system.

(b) Determine the linearized equations of motion about the equilibrium position $\theta = 0$, $\phi = 0$.

(c) Determine the natural frequencies and modes of vibration for small motions about equilibrium.

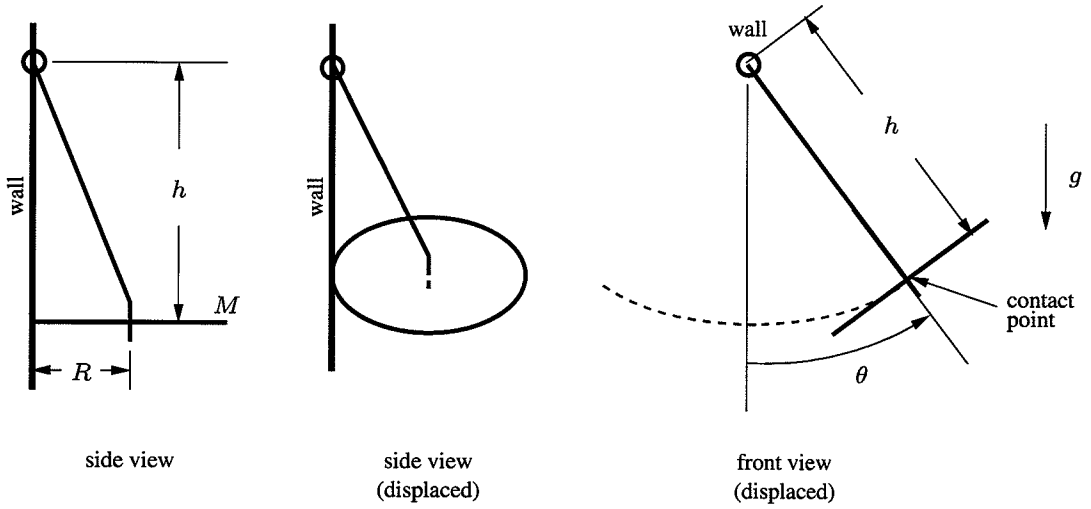


Problem 5

A disk of mass M and radius R rotates about a massless bent rod which is suspended from a frictionless pivot. The disk rolls without slip on the vertical wall, remaining perpendicular to it and tracing the circular arc shown. When $\theta = 0$, the plane of the disk is a vertical distance h below the pivot.

(a) Derive an equation of motion for the angle θ.

(b) Find the angular momentum of the system about the pivot point, and its time derivative.



side view

side view (displaced)

front view (displaced)