Summer 2008
PhD Qualifying Examination
Control Theory
(100 points; 3 hours)

1. (20 points) Consider the control system shown in Figure 1.
   (a) Compute the tracking error $E(s) = Y(s) - R(s)$. Determine values for K_1 and K_2 that guarantee closed-loop system stability.
   (b) Sketch the root locus and explain on the root locus plot how you would determine values of K_1 and K_2 to meet the following specifications:
      1. Percent overshoot $P.O. \leq 5\%$ due a reference unit step input $R(s) = 1/s$, and
      2. Settling time $T_s \leq 1$ sec due a reference unit step input $R(s) = 1/s$.
   (c) Describe the inherent conflict in control design associated with simultaneous disturbance rejection and noise attenuation. How is this conflict generally resolved in the classical approach to feedback control system design?
   (d) Using the results of (a)-(c), discuss how you would choose K_1 and K_2 to meet the design specifications while rejecting disturbances and attenuating noise as much as possible.

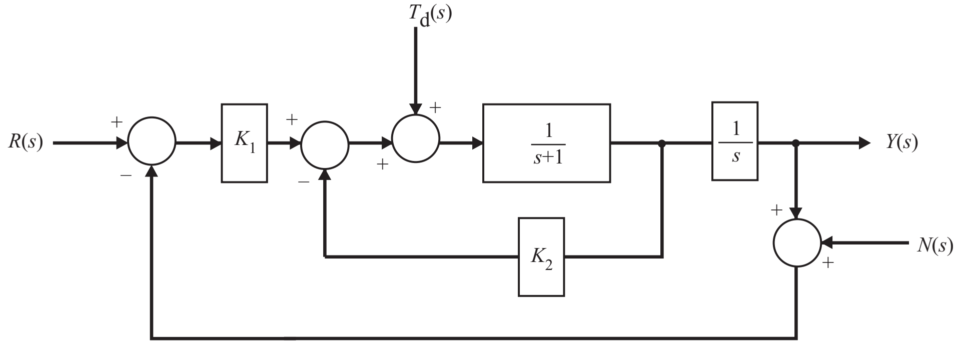

Figure 1: Multi-loop feedback system with reference, disturbance, and noise inputs.

2. (20 points) Consider a controllable and observable SISO linear system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu, \ y = C\boldsymbol{x}, \quad \text{with} \quad H(s) = C(sI - A)^{-1}B$$

   (a) If λ is not an eigenvalue of A, prove that there exists an initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_o$ such that the response of the system due to input $u(t) = e^{\lambda t}$, for $t \geq 0$ is described by $y(t) = H(\lambda)e^{\lambda t}$. Also, make sure you specifically discuss the implications of what happens when λ is a zero for $H(s)$.
   (b) Assume A has distinct eigenvalues and let λ be an eigenvalue of A. Show that there exists an initial state $\boldsymbol{x}_o$ such that the response of the system due to zero input (i.e., $u(t) \equiv 0$) is described by $y(t) = ke^{\lambda t}$ for all $t \geq 0$ for some finite real number constant k.
   (c) Now suppose the system is observable and stabilizable (but not controllable). Discuss in detail what impact, if any, these weaker hypotheses have on the results in part (b) above?

3. (15 points) Consider the system

$$\begin{aligned} G(s) &= \begin{bmatrix} \frac{s}{(s+2)^2} & \frac{s}{s+2} \\ \frac{s}{s+2} & \frac{s}{s+1} \end{bmatrix} \\ &= \begin{bmatrix} s(s+2) & s(s+1)^2 \\ s(s+1)^2 & s(s+2)(s+1) \end{bmatrix} \begin{bmatrix} (s+1)^2(s+2) & 0 \\ 0 & (s+1)^2(s+2) \end{bmatrix}^{-1} \\ &= \begin{bmatrix} (s+1)^2(s+2) & 0 \\ 0 & (s+2)(s+1) \end{bmatrix}^{-1} \begin{bmatrix} s(s+2) & s(s+1)^2 \\ s(s+1) & s(s+2) \end{bmatrix} \end{aligned}$$

(a) Determine the Smith form associated with the right MFD.

(b) Is the right MFD co-prime? Explain your answer.

(c) Is the left MFD co-prime? Explain your answer.

(d) Determine the Smith-Macmillan form associated with $G(s)$.

(e) Determine the poles and zeros of the MIMO system given by $G(s)$.

4. (10 points) Consider a unity feedback system, with plant

$$G(s) = \frac{1}{s(s+0.1)(s+100)},$$

subject to a proportional control law with gain K. Use a Bode diagram to determine the following quantities,

(a) The value of the controller gain that leads to a marginally stable system.

(b) The value of the gain that leads to a closed loop system with a dominant set of poles characterized by a damping ratio of "approximately" 0.5.

5. (20 points) Consider a linear system, $\dot{\bar{\boldsymbol{x}}} = A\bar{\boldsymbol{x}}$, with

$$A = \frac{1}{2}\begin{bmatrix} -7 & -1 \\ 1 & -9 \end{bmatrix}.$$

(a) Find the real Jordan form of A. Is the system diagonalizable?

(b) Compute the state transition matrix using the results from part (a).

6. (15 points) In the framework of *decentralized control* for MIMO systems, it is typically assumed that the plant is diagonal and controllers are designed independently for each diagonal element of the plant disregarding the effects of the off-diagonal elements. Obviously, the real plant is not completely decoupled, and the interactions due to the off-diagonal blocks can drive the closed-loop system to instability unless the controllers are designed to provide robust stability. Consider the two-input, two-output plant described by

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

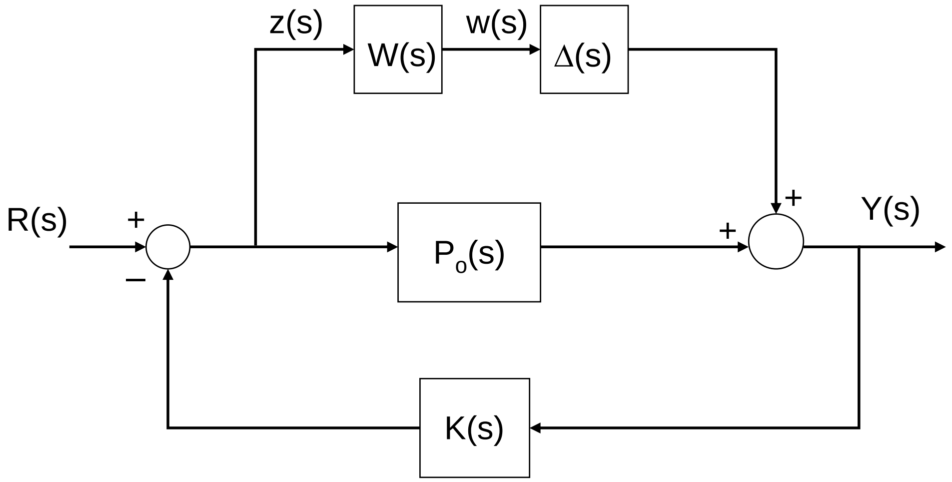

Figure 2: Block diagram for robust stability analysis via decentralized control.

wherein it is assumed that the transfer functions $P_{12}(s)$ and $P_{21}(s)$ are stable and "relatively small" compared to the diagonal elements $P_{11}(s)$ and $P_{22}(s)$. Suppose a decentralized controller $K(s) = \text{diag}(K_1(s), K_2(s))$ is designed to stabilize the nominal diagonal system $P_o(s) = \text{diag}(P_{11}(s), P_{22}(s))$ as shown in Figure 2.

Your tasks are as follows:

(a) Formulate this problem within the framework of robust stability analysis, i.e., frame the problem in the $M - \Delta$ form consistent with the block diagram shown in Figure 2.

(b) Assuming the off-diagonal elements are independent perturbations, derive the least conservative condition that ensures robust stability. Note that a generic answer such that $\mu(M) < 1$ isn't acceptable since this problem is analytically tractable to permit the establishment of an exact condition for robust stability.