Aerothermodynamics Ph. D. Qualifying Examination

June 2007

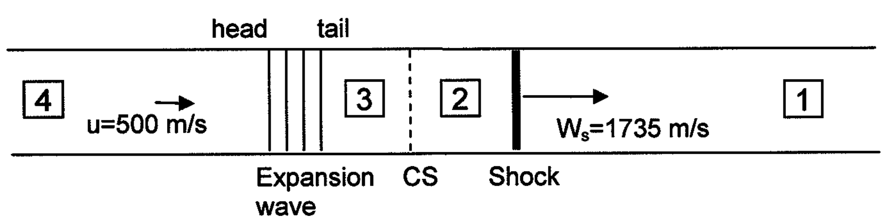
READ THESE INSTRUCTIONS.

*Answer all **five** questions. Please write on one side of the paper only and put your Code Number and appropriate question number on every sheet. **Begin each question on a separate sheet of paper.** To obtain complete credit, your work must be neat and your complete procedure shown. Draw neat sketches and list your assumptions. Ask for clarification if the meaning of a question is unclear to you.*

Problem 1: Consider the 1D unsteady flow in an expansion tube at the instant in time after the secondary shock has entered the acceleration section. In region 1 the secondary shock is propagating at a velocity of W_s=1735 m/s through stagnant air at T_1=300 K. This secondary shock was formed when a primary shock (not shown) burst a diaphragm at some point upstream. The primary shock induced the gas motion of 500 m/s to the right in region 4. When the secondary shock was formed, its acceleration produced a strong expansion wave **whose head is stationary in the laboratory frame of reference (i.e., its position relative to the tube remains constant in time)**. A contact surface (CS) separates regions 2 and 3. Assume γ=1.4 and that the air in the tube, before being processed by any wave systems, was stagnant at a temperature of 300 K.

(a) Sketch the shock, expansion wave and contact surface on a *t-x* diagram (make *t* the vertical axis).

(b) Determine the velocity (speed and direction) of the **tail** of the expansion wave.



Problem 2. Consider the one-dimensional, unsteady partial differential equation (PDE):

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

The following numerical scheme is proposed for solving the above PDE:

$$u_j^{n+1} = u_j^n - \frac{\gamma}{2}(u_{j+1}^n - u_{j-1}^n) + \frac{\gamma^2}{2}(u_{j+1}^n - 2u_j^n + u_{j-1}^n),$$

where $\gamma = c\Delta t / \Delta x$ and c is a constant.

a) By investigating the spatial discretization in the numerical scheme, determine the spatial accuracy of the method.

b) For the function u(x,t), write the series expansion in time.

c) Find the temporal accuracy of the numerical scheme by comparing the series expansion with the numerical scheme.

Problem 3. Consider and axial Couette flow in the gap between two concentric cylinders of radius R_1 and R_2 ($R_1 < R_2$). The inner cylinder moves axially with a velocity V_1 and the outer cylinder is stationary. Also the surface temperatures of the cylinders are kept fixed at T_w. Include the effect of frictional heating. Assuming constant fluid density and transport properties:

a) Determine the velocity and temperature profiles in the gap between the two cylinders.

b) Determine the net fluid mass flow rate between the cylinders.

c) Provide a rough (order-of-magnitude) estimate of the speed of the cylinder V_1 for which the peak temperature within the fluid due to frictional heating, is about twice the wall temperature. [Note: You can continue making the constant density and property assumption even though this is not consistent with the flow speeds at which frictional heating is important]

Problem 4. The polytropic efficiency ηpe for an expansion processs is defined by

$$\eta_{pe} \equiv \frac{dT}{dT_s}.$$

Here dT (< 0) is the temperature fall in an infinitesimal **irreversible** expansion, and dT_s is the temperature drop in the corresponding infinitesimal **reversible** expansion through the **same** pressure ratio.

a) Using Gibbs' equation and the above definition of η_{pe} prove that the pressure and temperature after an irreversible expansion from state 1→ state 2 with constant polytropic efficiency are related by: $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\eta_{pe}(\gamma-1)}{\gamma}}$.

b) The propellant in a rocket nozzle has a molar mass of 14 *kg/kmol* and a ratio of specific heats $\gamma = 1.20$. It expands adiabatically with constant polytropic efficiency η_{pe} = 0.98 from a stagnation temperature and pressure of 3600 *K* and 5.30 *MPa* respectively. The gas velocity at the exit plane of the nozzle is 3500 *m/s*. (i) Calculate the static pressure at the exit of the nozzle. (ii) If the nozzle has to pass a mass flow of 50 *kg/s* calculate the required throat area.

(The universal gas constant, $\hat{R} = 8314$ *J/kmol-K*)

Problem 5. Air is flowing along a flat plate at a Mach number of 0.8 with a stagnation temperature of 300 C. The wall temperature is 400 C.

(a) Is the heat transfer from the fluid to the plate or vice-versa? Explain.

(b) At what Mach number will there be zero heat transfer between the fluid and the plate?

(c) Sketch the temperature profile through the boundary layer for the above zero heat transfer case.