VELOCITY INCREMENT REQUIRED TO CIRCULARIZE AN ORBIT

Assume that a spacecraft is in a non-circular orbit with semi-major axis, a, and eccentricity, e. Suppose that it is required that the orbit be circularized at a radius, r, such $r_p \le r \le r_a$ using a single burn (we will consider the planar case only). The magnitude of velocity in the non-circular orbit at the radius where the orbit is to be circularized is given by

$$V = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

Recall that the orbit parameter (semi-latus rectum) of the orbit is given by the relation $p = a\left(1 - e^2\right)$. The true anomaly, f, is then given by $f = \cos^{-1}\left(\frac{p - r}{e\, r}\right)$.

(For this calculation, do not worry about obtaining the proper quadrant for f, use the value between zero and 180 degrees which is output by your calculator. The velocity increment required is the same for both possible values of the angle, f. If you need to know the proper value of f, recall that on the outbound half of the orbit, f lies between zero and 180 degrees whereas on the inbound half,, f lies between 180 degrees and 360 degrees.

At any point in the orbit, velocity is given by the vector equation

$$\overline{V} = \sqrt{\frac{\mu}{p}}\, e \sin f\ \overline{u}_R + \frac{\sqrt{\mu\, p}}{r}\ \overline{u}_H$$

where $\overline{u}_H$ is a unit vector in the horizontal direction, $\overline{u}_R$ is a unit vector in the radial (outward) direction, and μ is the gravitational parameter of the central body.

At any radius, the required circular velocity is given by $\overline{V}_C = \sqrt{\frac{\mu}{r}}\ \overline{u}_H$

The $\Delta\overline{V}$ required for circularization is given by $\Delta\overline{V} = \overline{V}_C - \overline{V}$, or in vector terminology, by

$$\Delta\overline{V} = \left(\sqrt{\frac{\mu}{r}} - \frac{\sqrt{\mu\, p}}{r} \right) \overline{u}_H - \sqrt{\frac{\mu}{p}}\, e \sin f\ \overline{u}_R$$

The magnitude of the required one-burn $\Delta\overline{V}$ is then given by

$$\left|\Delta\overline{V}\right| = \sqrt{\left(\sqrt{\frac{\mu}{r}} - \sqrt{\frac{\mu\, p}{r}} \right)^2 + \frac{\mu}{p}\ e^2 \sin^2 f}$$