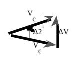
SINGLE IMPULSE MANEUVERS IN CIRCULAR ORBIT

Assume that a maneuverable spacecraft is in a circular orbit with a radius, r_C. A number of orbit changes can be made with a single impulse. Among them are:

1. SIMPLE PLANE CHANGE -- To execute a simple plane change (change inclination an amount Δi without changing the size or shape of the orbit), the required ΔV is given by



$$\Delta V^2 = 2\left(V_C^2 - V_C^2 \cos \Delta i\right)$$

$$\Delta V^2 = V_C \sqrt{2\left(1 - \cos \Delta i\right)}$$

Recall that the velocity in a circular orbit given by $V_C = \sqrt{\frac{\mu}{r_C}}$

and thus the required ΔV can be written as

$$\Delta V = V_C \sqrt{2\left(1 - \cos \Delta i\right)} = \sqrt{\frac{2\mu}{r_C}\left(1 - \cos \Delta i\right)}$$

2. MAKE ORBIT ELLIPTIC WITH PERIGEE AT CIRCULAR ORBIT

To make the new orbit elliptic with the perigee radius equal to th radius of the original circular orbit r_C, an apogee radius, r_A, must be specified. The semi-major axis, a, of the new orbit is then qiven by $a = \left(r_A + r_C\right)/2$ and the eccentricity, e, is given by $e = \frac{r_A - r_C}{2a}$.

The velocity at perigee, V_P, in the elliptic orbit is given by

$$V_P = \sqrt{\mu\left(\frac{2}{r_C} - \frac{1}{a}\right)}$$

and the required ΔV is then given by

$$\Delta V = V_P - V_C$$

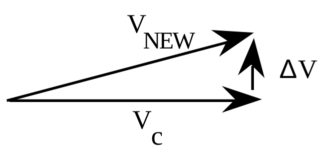
The burn is posigrade, tangent to the circular orbit, and is at the perigee of the elliptic orbit.

3. MAKE ORBIT ELLIPTIC WITH APOGEE AT CIRCULAR ORBIT

This case is very similar to the case above. In order to make the new orbit elliptic with the apogee radius equal to the radius of the original circular orbit, a desired perigee radius (r_P) must be specified. Then. the new semi-major axis is given by $a = \left(r_P + r_C\right)/2$, the eccentricity is given by $e = \dfrac{r_C - r_P}{2a}$, and the velocity in the new orbit at the burn point (at apogee) is given by $V_A = \sqrt{\mu \left(\dfrac{2}{r_C} - \dfrac{1}{a}\right)}$

The required ΔV is then given by $\Delta V = V_C - V_A$, where the ΔV is in the direction opposite to the velocity vector.

4. SIMPLE RADIAL BURNS -- Suppose that we want to see the effect of that a radial burn has on the size and shape of an orbit. Assume that we start in a circular orbit and apply a radial velocity impulse ΔV to the spacecraft. During the impulse, the radius, RO, will not change, but the new velocity will be given by



$$V_{NEW} = \sqrt{V_C^2 + \Delta V^2}$$

The new semi-major axis will be given by

$$a = \frac{\mu\ r_0}{2\ \mu - V_{NEW}^2\ r_0}$$

and the new flight path angle, γ , will be $\gamma = \sin^{-1}\left(\dfrac{\Delta V}{V_{NEW}}\right)$

Note that if ΔV is outward then $\gamma > 0$, and if ΔV is inward then $\gamma < 0$.

The new orbit parameter will be $p = \dfrac{\left(r_0\ V_{NEW} \cos\gamma\right)^2}{\mu}$, and the eccentricity wil1 be given by $e = \sqrt{1 - P/a}$.

The true anomaly at the burn point, f , is given $f = \cos^{-1}\left(\dfrac{p - r_0}{e\ r_0}\right)$

Note: when ΔV is outward, use $0 \le f \le 180^\circ$, and when ΔV is inward, use $180^\circ \le f \le 360^\circ$.