POINTING AT GEOSTATIONARY SATELLITES

As more of us become involved with satellite-relayed TV, it becomes increasingly important to know how to point toward a geostationary satellite. The general problem can be stated as follows:

> An antenna is located at a known point on the surface of the Earth (latitude, δ, and longitude, λ, are known). It is desired to point the antenna at a geostationary satellite located 22828 nmi. from the center of the Earth in the equatorial plane directly above longitude λ_G. Find the azimuth (angle measured east from north in the horizontal plane) and elevation (angle between the line of sight to the satellite and the local horizontal plane) of the line of sight (LOS) to the geostationary satellite.

There are several ways to attack this problem, and the simplest take advantage of the fact that the geometry is fixed relative to the Earth (the antenna site and the satellite are both "geostationary").

The vector from the center of the Earth to the antenna site, $\bar{r}_A$, is given by

$$\bar{r}_A = r_A \cos\lambda \cos\delta \; \bar{i} + r_A \sin\lambda \cos\delta \; \bar{j} + r_A \sin\delta \; \bar{k}$$

where

r_A is the distance between the antenna site and the center of the Earth, and

λ is the longitude of the antenna site (measured positive toward the east),

$$\left(0^\circ \le \lambda \le 360^\circ\right)$$

δ is the latitude of the antenna site (measured from the equatorial plane, positive to the north).

$$\left(-90^\circ \le \delta \le 90^\circ \right)$$

and $\bar{i}, \bar{j}, \bar{k}$, are unit vectors in Earth-fixed coordinates with $\vec{k}$ out the north pole, $\vec{i}$ in the equatorial plane pointing toward 0° longitude point and $\bar{j}$ such that ($\bar{i}, \bar{j}, \bar{k}$) is a right-handed coordinate triad ($\bar{j}$ points to the 90° east longitude point on the equator - in the Indian Ocean).

The vector from the center of the Earth to the geostationary satellite is

$$\bar{r}_G = r_G \cos\lambda_G \; \bar{i} + r_G \sin\lambda_G \; \bar{j}$$

Note that $\delta_G \equiv 0^\circ$ since all geostationary orbits must be equatorial $\left(i \equiv 0^\circ\right)$

The vector from the antenna to the satellite, $\bar{r}_{LOS}$, can be calculated from

$\bar{r}_{LOS} = \bar{r}_G - \bar{r}_A$.

Now that we have $\bar{r}_{LOS}$, all that is necessary is to express this vector in terms of local horizontal coordinates and then determine the azimuth and elevation angles. The coordinate transformation between the Earth fixed (i, j, k) axes and a local set $\left(\bar{i}', \bar{j}', \bar{k}'\right)$ such that $\bar{i}'$ points south, $\bar{j}'$ points east, and $\bar{k}'$ points up is given by

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \sin\delta & 0 & -\cos\delta \\ 0 & 1 & 0 \\ \cos\delta & 0 & \sin\delta \end{bmatrix} \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\lambda\sin\delta & \sin\lambda\sin\delta & -\cos\delta \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\lambda\cos\delta & \sin\lambda\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where (x, y, z) are components of any vector in the Earth fixed polar/equatorial coordinates and (x', y', z') are the components of the same vector in local coordinates. The vector we want to transform is $\bar{r}_{LOS}$. Let the components of $\bar{r}_{LOS}$ be denoted by $\bar{r}_{LOS} = a\bar{i} + b\bar{j} + c\bar{k}$, where

$$a = r_G \cos\lambda_G - r_A \cos\lambda\cos\delta$$

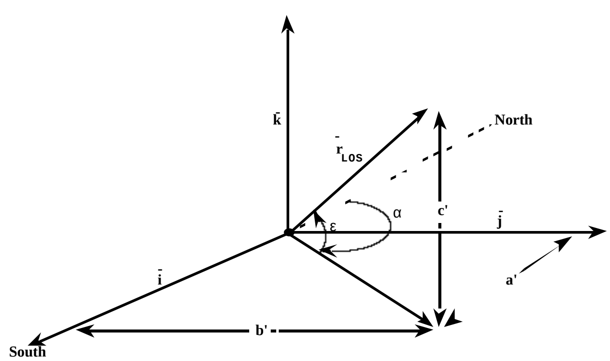
$$b = r_G \cos\lambda_G - r_A \sin\lambda\cos\delta$$

$$c = r_A \sin\delta$$

The components of the transformed vector, $\bar{r}_{LOS} = a'\,\bar{i}' + b'\,\bar{j}' + c'\,\bar{k}'$ are a', b', and c', and can be calculated using the transformation matrix. Specifically,

$$a' = a\cos\lambda\sin\delta + b\sin\lambda\sin\delta - c\cos\delta$$
$$b' = -a\sin\lambda + b\cos\lambda$$
$$c' = a\cos\lambda\cos\delta + b\sin\lambda\cos\delta + c\sin\delta$$

Now all that remains is to determine the azimuth and elevation angles given a', b', and c'. The geometry is as shown in the figure below:



The angle ε is the elevation and the angle α is the azimuth, α. An easy way to carry out the calculation is to rewrite $\bar{r}_{LOS}$ as

$$\bar{r}_{LOS} = \bar{A} + c'\bar{k}'$$

where

$$\bar{A} = a'\bar{i}' + b'\bar{j}'$$

The elevation, ε, can now be written as

$$\cos\,\varepsilon = \bar{A} \quad \bar{r}_{LOS} / \left(\left|\bar{A}\right| \;\; \left|\bar{r}_{LOS}\right| \right)$$

This uniquely determines ε since $-90^\circ <= \;\; <= 90^\circ$.

The azimuth, $\alpha = 90^\circ + \beta$, can be determined from

$$\cos\,\alpha = \frac{\left(-\bar{i}' \quad \bar{A}\right)}{\left|\bar{A}\right|}$$

and

$$-\bar{k}'\,\sin\,a = \frac{\left(-\bar{i}' \quad \bar{A}\right)}{\left|\bar{A}\right|}$$

Knowing both $\sin\alpha$ and $\cos\alpha$, we can uniquely determine $\left(0 \le \alpha < 360^\circ\right)$. [In FORTRAN use the ATAN2 function.]

SATELLITE POINTING PROBLEM

The latitude and longitude of Houston are $\left(30.0N,\ 95.5W\right)$. Determine pointing angles (azimuths and elevations) for a geostationary satellites located at longitudes between $80^\circ W$ and $140^\circ W$ (from $220^\circ E$ through $280^\circ E$) at 1° intervals.