7.7 Const. alt. accel. of ISA

$h = \text{Const}, \; P = \text{Const} \Rightarrow T(h,P) = \text{Const}. \qquad \gamma = 0$

V_0 and V_f are fixed

$$\dot{x} = V$$
$$\dot{V} = (g/W)(T - D)$$

$$V^* = \sqrt{\frac{2W}{\rho S C_L^*}} = \text{Const} \qquad D = \frac{W}{2E^*}\left(u^2 + \frac{1}{u^2}\right) \qquad u = \frac{V}{V^*}$$

$$\dot{x} = V^* u$$
$$V^* \dot{u} = \frac{g}{W}\left[T - \frac{W}{2E^*}\left(u^2 + \frac{1}{u^2}\right)\right]$$

$$\frac{dx}{du} = \frac{W}{g} \frac{V^{*2} u}{T - \frac{W}{2E^*}\left(u^2 + \frac{1}{u^2}\right)}$$

Define the nondimensional thrust as $\tau = \dfrac{T}{W/E^*}$

$$\frac{dx}{du} = \frac{E^* V^{*2}}{g} \quad \frac{u}{\tau - \frac{1}{2}(u^2 + 1/u^2)}$$

$$= \frac{2E^* V^{*2}}{g} \quad \frac{u^3}{2\tau u^2 - u^4 - 1}$$

$$dx = -\frac{2E^* V^{*2}}{g} \quad \frac{u^3 \, du}{u^4 - 2\tau u^2 + 1}$$

$$dx = -\frac{2E^* V^{*2}}{g} \quad \frac{1}{2} \quad \frac{u^2 \, du^2}{(u^2)^2 - 2\tau u^2 + 1} \qquad \frac{x \, dx}{x^2 - 2\tau x + 1}$$

$$X = x^2 - 2\tau x + 1 \qquad a = 1 \quad b = -2\tau \quad c = 1$$

$$\int \frac{x \, dx}{X} = \frac{1}{2a} \ln |X| - \frac{b}{2a} \int \frac{dx}{X}$$

7.7 Cont'd $b^2 - 4ac = 4\tau^2 - 4 = 4(\tau^2 - 1) > 0$

Integral tables

$$\int \frac{dx}{X} = \frac{1}{a(p-q)} \ln\left|\frac{x-p}{x-q}\right|$$

p, q roots of $x^2 - 2\tau x + 1 = 0$

$$p, q = \frac{2\tau \pm \sqrt{4\tau^2 - 4}}{2} = \tau \pm \sqrt{\tau^2 - 1}$$

$$x = u^2 \qquad p = u_1^2 \qquad q = u_2^2$$

$$u_1 = \sqrt{\tau + \sqrt{\tau^2 - 1}} \qquad u_2 = \sqrt{\tau - \sqrt{\tau^2 - 1}}$$

$$\int \frac{x\,dx}{X} = \frac{1}{2} \ln\left|u^4 - 2\tau u^2 + 1\right|$$

$$+ \tau \frac{1}{u_1^2 - u_2^2} \ln\left|\frac{u^2 - u_1^2}{u^2 - u_2^2}\right|$$

$$= \frac{1}{2} \ln\left|(u^2 - u_1^2)(u^2 - u_2^2)\right| + \frac{\tau}{u_1^2 - u_2^2} \ln\left|\frac{u^2 - u_1^2}{u^2 - u_2^2}\right|$$

$$u_1^2 + u_2^2 = \tau + \sqrt{\tau^2 - 1} + \tau - \sqrt{\tau^2 - 1} = 2\tau$$

$$= \frac{1}{2} \ln\left|(u^2 - u_1^2)(u^2 - u_2^2)\right| + \frac{1}{2}\frac{u_1^2 + u_2^2}{u_1^2 - u_2^2} \ln\left|\frac{u^2 - u_1^2}{u^2 - u_2^2}\right|$$

$$= \frac{1}{2}\left[\ln(u^2 - u_1^2) + \ln(u^2 - u_2^2)\right] + \frac{1}{2}\frac{u_1^2 + u_2^2}{u_1^2 - u_2^2}\left[\ln (u_1^2 - u^2) - \ln (u^2 - u_2^2)\right]$$

$$= \frac{1}{2}\frac{1}{u_1^2 - u_2^2}\Big\{(u_1^2 - u_2^2)\ln(u_1^2 - u^2) + (u_1^2 - u_2^2)\ln(u^2 - u_2^2)$$
$$+ (u_1^2 + u_2^2)\ln(u_1^2 - u^2) - (u_1^2 + u_2^2)\ln(u^2 - u_2^2)\Big\}$$

$$= \frac{1}{2}\frac{1}{u_1^2 - u_2^2}\left\{2u_1^2 \ln(u_1^2 - u^2) - 2u_2^2 \ln(u^2 - u_2^2)\right\}$$

$$\int \frac{x\,dx}{X} = \frac{1}{u_1^2 - u_2^2}\left[u_1^2 \ln(u_1^2 - u^2) - u_2^2 \ln(u^2 - u_2^2)\right]$$

7.7 Cont'd

$$x_f - x_0 = -\frac{E^* V^{*2}}{g}\left[A(u_f) - A(u_0) \right]$$

$$A(u) = \frac{1}{u_1^2 - u_2^2}\left[u_1^2 \ln(u_1^2 - u^2) - u_2^2 \ln(u^2 - u_2^2) \right]$$

$$u_1 = \sqrt{\tau + \sqrt{\tau^2 - 1}} \qquad u_2 = \sqrt{\tau - \sqrt{\tau^2 - 1}}$$