7.4 Nonsteady Glide

$T=0$, ISA $\Rightarrow C_D = C_{D_0} + K C_L^2$, exponential atmosphere

$$\left.\begin{aligned} \dot{x} &= V \\ \dot{h} &= V\gamma \\ \dot{V} &= (g/W)(-D - W\gamma) \\ 0 &= L - W \end{aligned}\right\} \Rightarrow \frac{dE_s}{dt} = P_s = \frac{-D(h,V,W)V}{W} < 0$$

$$\frac{dx}{dE_s} = \frac{V}{-\frac{DV}{W}} = -\frac{W}{D}$$

$$x_f - x_0 = -\int_{E_{s0}}^{E_{sf}} \frac{W}{D}\, dE_s = \int_{E_{sf}}^{E_{s0}} \frac{W}{D(h,V,W)}\, dE_s$$

To maximize distance, we want max drag as

$$\left.\frac{\partial D}{\partial V}\right|_{E_s = \text{Const}} = 0$$

$$D = C_{D_0} S \bar{q} + \frac{K W^2}{\bar{q} S} \qquad \bar{q} = \frac{1}{2}\rho V^2$$

$$\rho = \rho_s e^{\left(-\frac{h}{\lambda}\right)}, \quad h = E_s - \frac{V^2}{2g} \qquad \rho = \rho_s\, e^{-\frac{E_s - \frac{V^2}{2g}}{\lambda}}$$

$$\left.\frac{\partial D}{\partial V}\right|_{E_s = \text{Const}} = \left(C_{D_0} S - \frac{K W^2}{\bar{q}^2 S}\right) \left.\frac{\partial \bar{q}}{\partial V}\right|_{E_s = \text{Const}} = 0$$

$$\frac{\partial \bar{q}}{\partial V} = \frac{1}{2}\frac{\partial \rho}{\partial V} V^2 + \frac{1}{2}\rho\, 2V \qquad \frac{\partial \rho}{\partial V} = \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial V} \qquad \frac{\partial \rho}{\partial h} = -\frac{\rho}{\lambda}$$

$$\frac{\partial \bar{q}}{\partial V} = \frac{1}{2}\left(-\frac{\rho}{\lambda}\right)\left(-\frac{2V}{2g}\right) V^2 + \rho V$$

$$\frac{\partial \bar{q}}{\partial V} = \rho V \left[\frac{V^2}{\lambda g} + 1\right] \Rightarrow \left.\frac{\partial \bar{q}}{\partial V}\right|_{E_s = \text{Const}} \neq 0$$

7.4 Cont'd

min drag occurs when

$$C_{D_0} S - \frac{K W^2}{\bar{q}^2 S} = 0$$

$$\bar{q} = \sqrt{\frac{K}{C_{D0}}}\ \frac{W}{S} = \frac{W}{S\, C_L^*}$$

$$\frac{1}{2}\, \rho_s\, e^{-\left(\frac{E_s - \frac{V^2}{2g}}{\lambda}\right)} V^2 = \frac{W}{S C_L^*}$$

solve for $V(E_s)$

The problem of maximizing the time follows the same procedure.