7.1 approximate integration for energy climb

$$t_f - t_o = \int_{E_{s_o}}^{E_{s_f}} \frac{dE_s}{P_s(E_s, V)}$$

$$P_s(E_s, V) = \left[\frac{T(h,V,P) - D(h,V,L)}{W}\right] V \qquad P, W \text{ const.}$$

$$h = E_s - \frac{V^2}{2g}$$

$$t_f - t_o = \sum_{k=1}^{n} \int_{E_{s_k}}^{E_{s_{k+1}}} \frac{dE_s}{P_s(E_s, V)}$$

over this energy interval, assume that maximum P_s is linear in E_s

$$P_s = P_{s_k} + \frac{P_{s_{k+1}} - P_{s_k}}{E_{s_{k+1}} - E_{s_k}} (E_s - E_{s_k})$$

$$\int_{E_{s_k}}^{E_{s_{k+1}}} \frac{dE_s}{P_{s_k} + \dfrac{P_{s_{k+1}} - P_{s_k}}{E_{s_{k+1}} - E_{s_k}} (E_s - E_{s_k})}$$

$$= \frac{E_{s_{k+1}} - E_{s_k}}{P_{s_{k+1}} - P_{s_k}} \ln \left[P_{s_k} + \frac{P_{s_{k+1}} - P_{s_k}}{E_{s_{k+1}} - E_{s_k}} (E_s - E_{s_k}) \right]_{E_{s_k}}^{E_{s_{k+1}}}$$

$$= \frac{E_{s_{k+1}} - E_{s_k}}{P_{s_{k+1}} - P_{s_k}} \ln \left(\frac{P_{s_{k+1}}}{P_{s_k}} \right)$$

$$(t_f - t_o)_{max} = \sum_{k=1}^{n} \frac{E_{s_{k+1}} - E_{s_k}}{P_{s_{k+1}} - P_{s_k}} \ln \left(\frac{P_{s_{k+1}}}{P_{s_k}} \right)$$

where P_{s_k} is the max value of P_s at E_{s_k}