5.9 Climb - cruise

P, C_L Const $\qquad C_D(C_L), E(C_L) \Rightarrow C_D$ const. E const.

ISA stratosphere $\qquad T = T_t\left(\frac{\rho}{\rho_t}\right) \qquad C = C_t$

$a=1 \quad b=0$

$\varphi S + W\gamma \ll T$ or D

$$T - D = 0 \qquad L - W = 0$$

$$T_t \frac{\rho}{\rho_t} - \frac{1}{2} C_{D_0} \rho S V^2 - \frac{2KW^2}{\rho S V^2} = 0$$

$$\frac{1}{2} C_L \rho S V^2 = W$$

$$\frac{1}{2}\rho S V^2 = \frac{W}{C_L}$$

$$T_t \frac{\rho}{\rho_t} - C_{D_0}\frac{W}{C_L} - \frac{KW^2}{\frac{W}{C_L}} = 0$$

$$\frac{T_t}{\rho_t}\rho = \frac{C_{D_0}}{C_L}W + KC_L W$$

$$= \frac{C_{D_0} + KC_L^2}{C_L}W = \frac{W}{E}$$

$$\boxed{\rho = \frac{\rho_t}{T_t}\frac{W}{E}}$$

$$V^2 = \frac{2W}{\rho S C_L} = \frac{2W}{\frac{\rho_t}{T_t}\frac{\cancel{W}}{E} S C_L} = \frac{2T_t E}{\rho_t S C_L} \qquad V = \text{Const}$$

$$\boxed{V = \sqrt{\frac{2T_t E}{\rho_t S C_L}}}$$

5.9 Cont'd

$$\frac{dx}{dW} = -\frac{V}{CT}$$

$$x_f - x_0 = \int_{W_f}^{W_0} \frac{V}{CT}\, dW$$

V, C const. $\quad T = T_t \frac{\rho}{\rho_t} = \frac{\cancel{T_t}}{\cancel{\rho_t}} \frac{\cancel{\rho_t}}{\cancel{T_t}} \frac{W}{E}$

$$x_f - x_0 = \frac{V}{C}\int_{W_f}^{W_0} E \frac{dW}{W}$$

$$= \frac{EV}{C} \ln \Big|_{W_f}^{W_0}$$

$$= \frac{EV}{C} \ln \frac{W_0}{W_f}$$

$$= \frac{E}{C}\sqrt{\frac{2 T_t E}{\rho_t S C_L}} \ln \frac{W_0}{W_f}$$

$$= \frac{E^{3/2}}{C\, C_L^{1/2}} \sqrt{\frac{2T_t}{\rho_t S}} \ln \frac{W_0}{W_f}$$

$$x_f - x_0 \sim \frac{\left(\frac{C_L}{C_{D_0} + K C_L^2}\right)^{3/2}}{C_L^{1/2}} = \frac{C_L}{(C_{D_0} + K C_L^2)^{3/2}}$$

$$\frac{d\Delta x}{dC_L} = \frac{(C_{D_0} + K C_L^2)^{3/2}(1) - C_L \frac{3}{2}(C_{D_0} + K C_L^2)^{1/2} 2K C_L}{(C_{D_0} + K C_L^2)^{3/2}} = 0$$

$$(C_{D_0} + K C_L^2)^{3/2} - 3K C_L^2 (C_{D_0} + K C_L^2)^{1/2} = 0$$

$$C_{D_0} + K C_L^2 - 3K C_L^2 = 0$$

$$C_{D_0} - 2K C_L^2 = 0$$

$$C_L = \sqrt{\frac{C_{D_0}}{2K}} = \frac{C_L^*}{\sqrt{2}}$$

$$E = \frac{C_L}{C_{D_0} + K C_L^2} = \frac{\frac{C_L^*}{\sqrt{2}}}{C_{D_0} + K \frac{C_L^{*2}}{2}} = \frac{\frac{\sqrt{\frac{C_{D_0}}{K}}}{\sqrt{2}}}{C_{D_0} + \frac{K}{2}\frac{C_{D_0}}{K}} = \frac{\sqrt{\frac{C_{D_0}}{K}}}{C_{D_0} \frac{3}{2}\sqrt{2}}$$

5.9 Cont'd

SBJ $\rho_t = .000706$ $T_t = 1420$ $W_0 = 12{,}000$ $W_f = 10{,}000$

$$\rho_0 = \frac{\rho_t}{T_t}\frac{W}{E}$$

$$E = \frac{C_L}{C_D} \quad \text{for } C_L = \frac{C_L^*}{\sqrt{2}} \quad E = \frac{\frac{C_L^*}{\sqrt{2}}}{C_{D0} + K\frac{C_L^{*2}}{2}} = \frac{\frac{1}{\sqrt{2}}\sqrt{\frac{C_{D0}}{K}}}{C_{D0} + \frac{C_{D0}}{2}}$$

$$E = \frac{\frac{1}{\sqrt{2}}\sqrt{\frac{C_{D0}}{K}}}{\frac{3}{2}C_{D0}\quad\frac{2}{2}} = \frac{\frac{1}{\sqrt{2}}}{\frac{3}{4}} \quad E = \frac{4}{3\sqrt{2}}E^* = .9428 \quad 12.2 = 11.5$$

$$C_L = \frac{C_L^*}{\sqrt{2}} = \frac{.561}{1.414} = .397$$

$$\rho_0 = \frac{.000706}{1420}\frac{12{,}000}{11.5} = .000519 \qquad h_0 =$$

42,000	5.31 E-4	
	5.19 E-4	
43,000	5.07 E-4	

$\frac{.12}{.24}\,1000 = 500$ ft $\quad h_0 = 42{,}500$ ft

$$\rho_f = \frac{.000706}{1420}\frac{10{,}000}{11.5} = .000432 \qquad h_f =$$

46000	4.39 E-4
	4.32 E-4
47000	4.18 E-4

$\frac{.07}{.21}\,1000 = 333 \quad h_f = 46{,}333$ ft

$$V = \sqrt{\frac{2T_tE}{\rho_t S C_L}} = \sqrt{\frac{2(1420)\,11.5}{.000706\,(232)\,.397}} = 709 \text{ ft/s.}$$

$$X_f - X_0 = \frac{E^{3/2}}{C\;C_L^{1/2}}\sqrt{\frac{2T_t}{\rho_t S}}\;\ln\frac{W_0}{W_f}$$

$$= \frac{\overset{39.0}{(11.5)^{3/2}}}{\underset{.0002065}{\frac{1.18}{3600}}(.397)^{1/2}}\sqrt{\frac{2\,(1420)}{.000706\,(232)}}\;\ln\frac{12{,}000}{10{,}000}$$

$$= 188{,}800 \quad 131.7 \quad .1823 \div 5280 = 858.8 \text{ mi}$$

5,9 Cont'd.

Max dist C_L = Const h = Const (42,500 ft) ④

$$C_L = \frac{C_L^*}{\sqrt{3}} = \frac{.561}{1.732} = .323$$

$$E = \frac{.323}{.023 + .073\,(.323)^2} = 10.59$$

$$x_f - x_0 = \frac{2E}{C}\sqrt{\frac{2}{\rho S \cdot C_L}}\left(\sqrt{W_0} - \sqrt{W_f}\right)$$

$$= \frac{2(10.58)}{\frac{1.18}{3600}}\sqrt{\frac{2}{.000519\,(232)\;.323}}\left(\sqrt{12{,}000} - \sqrt{10{,}000}\right)$$

$$109.54 \quad - \quad 100$$

$$= 64{,}560 \quad 7.171 \quad 9.54 \div 5280 = 836 \text{ mi}$$

$$\frac{859 - 836}{836} \quad .0275$$

2.75 % more range for 2,000 lb of fuel $\frac{W_{fuel}}{W_0} = .17$