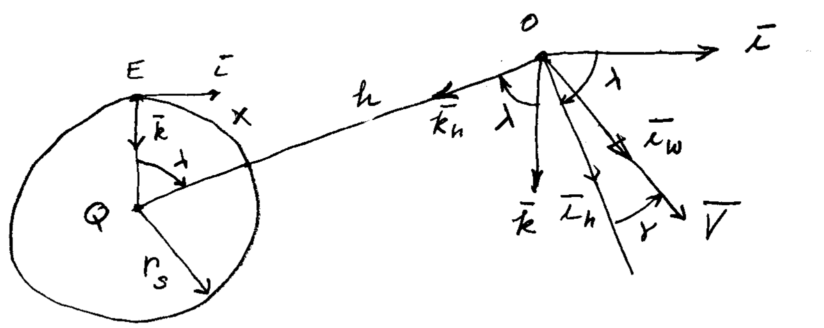
2.10 Equations of motion for flight in a great circle plane over a spherical earth.



$$\frac{d\overline{EO}}{dt} = \overline{V}$$

$$\overline{EO} = \overline{EQ} + \overline{QO} \qquad \dot{\overline{EQ}} = 0 \qquad \dot{\overline{EO}} = \dot{\overline{QO}}$$

$$\overline{QO} = -(r_s + h)\,\overline{k}_h$$

$$\frac{d\overline{EO}}{dt} = -\dot{h}\,\overline{k}_h - (r_s + h)\,\dot{\overline{k}}_h$$

$$\overline{k}_h = -\sin\lambda\,\overline{i} + \cos\lambda\,\overline{k}$$

$$\dot{\overline{k}}_h = -\cos\lambda\,\dot{\lambda}\,\overline{i} - \sin\lambda\,\dot{\lambda}\,\overline{k}$$

$$= -\dot{\lambda}\,(\cos\lambda\,\overline{i} + \sin\lambda\,\overline{k}) = -\dot{\lambda}\,\overline{i}_h$$

$$\dot{\overline{k}}_h = -\dot{\lambda}\,\overline{i}_h$$

$$\frac{d\overline{EO}}{dt} = (r_s + h)\,\dot{\lambda}\,\overline{i}_h - \dot{h}\,\overline{k}_h$$

$$\overline{V} = V\cos\gamma\,\overline{i}_h - V\sin\gamma\,\overline{k}_h$$

$$* \qquad \therefore \quad \dot{\lambda} = \frac{V\cos\gamma}{r_s + h}, \qquad \dot{h} = V\sin\gamma$$

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x is a curvalinear coordinate along the surface of the earth.

$\therefore \quad r_s \lambda = x \qquad \lambda = \frac{x}{r_s}$

$\dot{x} = \frac{r_s V \cos\gamma}{r_s + h}, \qquad \dot{h} = V \sin\gamma$

$\bar{V} = V \bar{i}_w$

$\bar{a} = \dot{V} \bar{i}_w + V \dot{\bar{i}}_w$

$\bar{i}_w = \cos(\lambda - \gamma)\, \bar{i} + \sin(\lambda - \gamma)\, \bar{k}$

$\dot{\bar{i}}_w = -\sin(\lambda - \gamma)(\dot{\lambda} - \dot{\gamma})\, \bar{i} + \cos(\lambda - \gamma)(\dot{\lambda} - \dot{\gamma})\, \bar{k}$

$= (\dot{\lambda} - \dot{\gamma})\left(-\sin(\lambda - \gamma)\, \bar{i} + \cos(\lambda - \gamma)\, \bar{k}\right)$

$\dot{\bar{i}}_w = (\dot{\lambda} - \dot{\gamma})\, \bar{k}_w$

$\bar{a} = \dot{V} \bar{i}_w + V(\dot{\lambda} - \dot{\gamma})\, \bar{k}_w$

$\bar{F} = m\bar{a}$ see Fig 2.2 for forces

$T \cos\varepsilon - D - W \sin\gamma = m \dot{V}$

$-T \sin\varepsilon - L + W \cos\gamma = m V(\dot{\lambda} - \dot{\gamma})$

$\dot{V} = \frac{g}{W}(T \cos\varepsilon - D - W \sin\gamma)$

$\dot{\gamma} = \frac{g}{WV}\left(T \sin\varepsilon + L - W \cos\gamma\right) + \dot{\lambda}$

2.10 Cont'd

$$\dot{V} = \frac{g}{W}\left(T\cos\varepsilon - D - W\sin\gamma\right)$$

*

$$\dot{\gamma} = \frac{g}{WV}\left(T\sin\varepsilon + L - W\cos\gamma\right) + \frac{V\cos\gamma}{r_s + h}$$

Also,

$$\dot{W} = -CT, \qquad g = g_s\left(\frac{r_s}{r_s + h}\right)^2 \quad \text{(inverse-square law)}$$