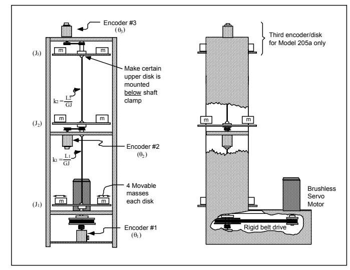
Identification of the Torsional Plant Parameters

Lecture 7

Torsional Apparatus



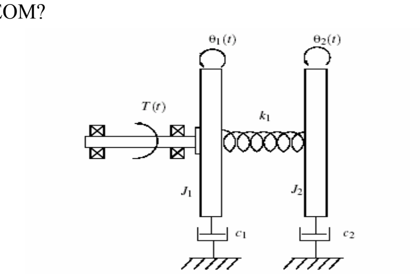
Dynamic Equivalence

- It is important to recognize the dynamic equivalence between rotational and linear dynamic systems.

Rotational System Parameters	**Linear System Parameters**
Inertia, J	Mass, m
Torsional Spring, K	Rectlinear Spring, K
Rotary Damping, c	Rectlinear Damping, c

Dynamic Equivalence – cont.

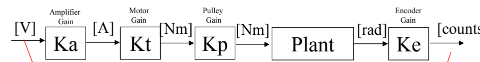
EOM?



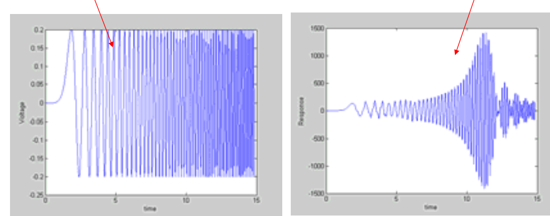
Today's Experiment

- Identify the Parameters of the Torsional Plant using least squares and compare with the same technique that we used for the linear plant.
- Run an open loop sine sweep input and collect output (angular displacement and voltage applied to the plant)
- Manually displace Disks using the same procedure as the rectlinear plant.

Today's Experiment - cont

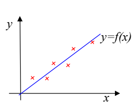


$\Rightarrow$ Can form data (V_i, θ_i)



LS Identification

- Least Square Curve Fitting



$\Rightarrow$Given: (x_i, y_i)
$\Rightarrow$Determine $y = Kx$, i.e, K which minimize the square sum of the erros

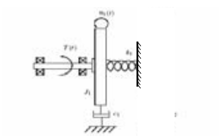
- Form $\Rightarrow y = Kx + e$
- Minimize $\Rightarrow J(e) = e^Te = [y-Kx]^T[y-Kx]$
- Solution $\Rightarrow K_{est} = (x^Tx)^{-1}x^Ty$

LS Identification – cont (1)

- LS Parameter Identification
  - Transform Differential Equations in a Linear Algebraic Equation
  - Form $\Rightarrow Y = F\theta + E$
  - Choose θ which minimizes the function
    - $J(\theta) = E^TE = [Y - F\theta]^T[Y - F\theta]$
    - $\theta_{LS} = [F^T F]^{-1}F^TY$

LS Identification – cont (2)

- Example: Determination of the Motor + Amplifier Gains
- Consider the following plant setup (1 DOF – Disks 2 and 3 are clamped)



EOM:

$$\ddot{\theta}_1 = -\frac{k_1}{J_1} - \frac{c_1}{J_1} + \frac{1}{J_1}T$$

LS Identification – cont (3)

- Let $T = KV$ be the torque applied to the plant through the motor and amplifier box (open loop)
- We can write

$$\ddot{\theta}_1 = -\frac{k_1}{J_1} - \frac{c_1}{J_1} + \frac{1}{J_1}KV \Rightarrow \ddot{\theta}_1 = \begin{bmatrix} -\theta_1 & -\dot{\theta}_1 & V \end{bmatrix} \begin{bmatrix} \frac{k_1}{J_1} \\ \frac{c_1}{J_1} \\ \frac{K}{J_1} \end{bmatrix}$$

$$\ddot{\theta}_1 = \begin{bmatrix} -\theta_1 & -\dot{\theta}_1 & V \end{bmatrix} P_0$$

P_0

LS Identification – cont (4)

- Assume *N* data are taken from the experiment
  - θ_1(k) and V(k) are measure directly
  - Approximate angular velocity and acceleration by central differences

$$\dot{\theta}_1(k) = \frac{\theta_1(k+1) - \theta_1(k-1)}{2T}$$

$$\ddot{\theta}_1(k) = \frac{\theta_1(k+1) - 2\theta_1(k) + \theta_1(k-1)}{T^2}$$

- We can define the estimation error as

$$e(k) = \ddot{\theta}_1(k) - \ddot{\theta}_1(k)_{estimated} = \ddot{\theta}_1 - \begin{bmatrix} -\theta_1(k) & -\dot{\theta}_1(k) & V \end{bmatrix} P_0$$

LS Identification – cont (5)

- The previous equation can be stacked up from k=2 to N-1 and we have

$$Y = \Phi P_0 + e \Rightarrow e = Y - \Phi P \quad \Rightarrow$$

$$e = \begin{bmatrix} e(2) & e(3) & \cdots & e(N-1) \end{bmatrix}^T$$

$$Y = \begin{bmatrix} \ddot{\theta}_1(2) & \ddot{\theta}_1(3) & \cdots & \ddot{\theta}_1(N-1) \end{bmatrix}^T$$

$$\Phi = \begin{bmatrix} -\theta_1(2) & -\dot{\theta}_1(2) & V(2) \\ -\theta_1(3) & -\dot{\theta}_1(3) & V(3) \\ \vdots & \vdots & \vdots \\ -\theta_1(N-1) & -\dot{\theta}_1(N-1) & V(N-1) \end{bmatrix}$$

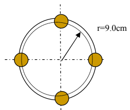
- We can form the LS problem as
  - min $\boldsymbol{J(e) = e^T e}$
  - $\boldsymbol{P_0 = (\Phi^T \Phi)^{-1} \Phi^T Y}$
- How can we compute *K* (Motor + Amplifier Gain)?
  - Because the way P_0 was defined, we can only estimate the ratios given. So how can we obtain the absolute values ?

LS Identification – cont (6)

- Take data from another experiment with different inertia, and compute new P, let's say $P_{w.}$

$$P_w = \left[\frac{k_1}{J_1 + J_w} \quad \frac{c_1}{J_1 + J_w} \quad \frac{K}{J_1 + J_w} \right]^T$$

- Setup the Disk #1 with four brass weights



$$J_w = n\left(\frac{1}{2}mr^2 + md^2\right)$$

Where:
n= number of weights = 4
m = mass of each weight = 500 g
r = radius of the weight = 2.5 cm
D = distance from the CL of the shat to the weight = 9.0 cm

LS Identification – cont (7)

- Finally,
  - We can compare P_0 and P_w and obtain the value for *K*
  - $P_0(3) = K/J_1$
  - $P_w(3) = K/(J_1 + J_w)$
- Estimate the rest of the plant parameters using the apparatus configured with all three disks free ⇒ Lab Report
  - Derive the EOM for the 3 disks free
  - Derive the LS equations

LS Identification

- Improvements on the LS
  - Recursive Least-Squares System ID.
    - The vector θ can be recalculated with each set of data as it arrives
  - Stochastic LS ⇒ data might be subject to random effects, so we need to include some probability functions
    - ARMA
- Other Methods
  - Nonparametric Models ⇒frequency response methods or Bode Plots

Today's Lab + Lab Report

- Run the Demo to familiarize with the Torsional Apparatus
- Obtain the Torsional Parameters using the LS and 2nd. Order prototype techniques (recall rectilinear plant)
- Save the Raw Data (Do not save plot)
- How can we read the file in MatLab?
  - Change extension to .m (or save as), say, *ident1.m*
  - Comment first line
  - Assign variable name
    - Data1 = […

      …]; dfsd
      - » Name of the file, say, *indent1*
      - » *Data1 becomes available in the workspace and can manipulate as a matrix*