

# Department of Aerospace Engineering and Engineering Mechanics



PhD Written Qualifying Examination  
Friday, June 8, 2012  
9:00 am – 12 noon

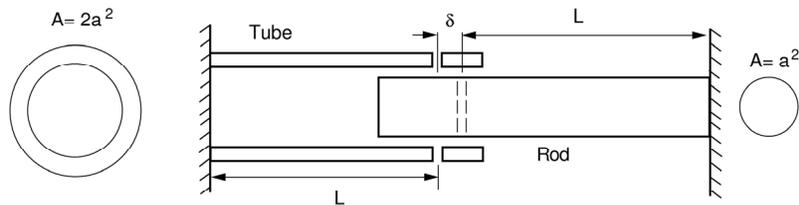
Answer all questions

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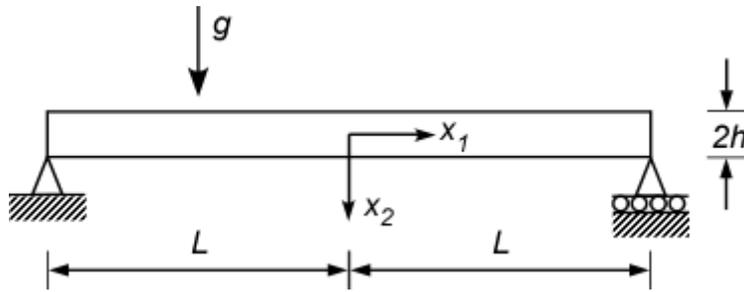
THE UNIVERSITY OF TEXAS AT AUSTIN

1. A pipe (modulus of elasticity  $2E$ , yield strength  $\sigma_0$ , coefficient of thermal expansion  $\alpha$ ) has been inserted snugly around a rod (modulus of elasticity  $E$ , yield strength  $1.5\sigma_0$ , coefficient of thermal expansion  $2\alpha$ ), but the holes for the connecting pin do not line up; there is a gap  $\delta$  (see figure). The pipe has a cross-sectional area  $2a^2$  and length  $L = 50a$ ; the rod has a cross-sectional area  $a^2$  and length  $L = 50a$ . The user applies forces on the pipe and the rod to close the gap, inserts a pin and then removes the forces.

- (a) What is the maximum allowable gap  $\delta$  if neither material should yield?
- (b) If the gap is set at one half of the value determined in part (a), what temperature change  $\Delta T$  will cause yield?
- (c) Given a gap, for what  $\Delta T$  will the pipes be stress free?



2. A heavy beam of density  $\rho$  is loaded under its own weight. It is simply supported as shown below. In attempting an elasticity solution for the problem, the stress function  $\phi_1 = \frac{1}{6}\rho g(3bx_1^2 + x_2^3)$  is proposed.
- Determine and comment on the applicability of this stress function (see part (b) for a hint).
  - In order to progress with the solution, consider the related problem of the same beam subject to a uniformly distributed lateral load  $q_0$ . A potential stress function for this problem is  $\phi_2 = \frac{A_2x_1^2}{2} + \frac{1}{2}B_3x_1^2x_2 + \frac{D_3x_2^3}{6} + \frac{1}{6}D_5x_1^2x_2^3 + \frac{F_5x_2^5}{20}$ . Establish the applicability of this related problem; in doing so, it will be helpful to consider relaxing the boundary conditions on  $x_1 = \pm L$ .
  - Determine the stresses in the beam due to its weight.



Equations:

$$\sigma_{11} = \frac{\partial^2 \phi}{\partial x_2^2} + V, \quad \sigma_{22} = \frac{\partial^2 \phi}{\partial x_1^2} + V, \quad \sigma_{12} = -\frac{\partial^2 \phi}{\partial x_1 \partial x_2}$$

$$f_1 = -\frac{\partial V}{\partial x_1}, \quad f_2 = -\frac{\partial V}{\partial x_2}$$

$$\nabla^4 \phi = -(1-\nu)\nabla^2 V$$

3. For orthotropic materials, Hooke's law is written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix}$$

where  $s_{ij}$  are compliances.

- a) Consider plane stress problems in the  $x$ - $y$  plane. Show that the Airy stress function  $\phi(x,y)$  must satisfy the following equation:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\rho\sqrt{\lambda} \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \lambda \frac{\partial^4 \phi}{\partial y^4} = 0$$

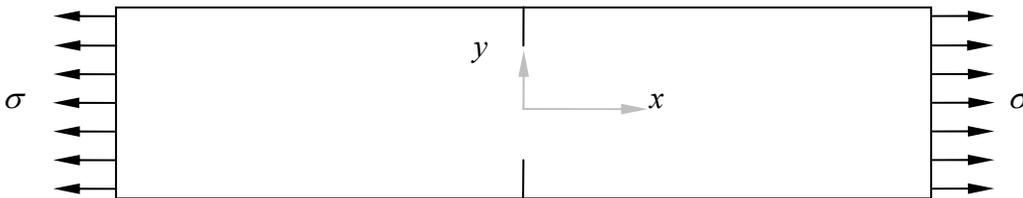
where

$$\lambda = \frac{s_{11}}{s_{22}}, \quad \rho = \frac{2s_{12} + s_{66}}{2\sqrt{s_{11}s_{22}}}$$

- b) Argue that the governing equation is of the same form for plane strain problems but  $s_{ij}$  in a) should be replaced by

$$s'_{ij} = s_{ij} - \frac{s_{i3}s_{j3}}{s_{33}}$$

- c) For traction-prescribed plane problems on simply-connected domains, argue that the in-plane stresses depend on elastic constants only through  $\rho$  and  $\lambda$ .
- d) Consider edge cracks (slits with no thickness) in an infinite strip subjected to a tensile load as illustrated in the following figure. Show that stress component  $\sigma_x$  is independent of  $\lambda$ . (Hint. Examine the governing equation and boundary condition in coordinates  $(\xi, y)$ , where  $\xi = \lambda^{1/4}x$ ).



4. A rod of initial length  $L$ , uniform cross-sectional area  $A$ , and mass density  $\rho$  is rotating in a plane at a constant angular velocity  $\Omega$ . Given that the material obeys the stress-strain response  $\varepsilon = C\sigma^n$  ( $C$  and  $n$  are constants), calculate the elongation of the rod (Hint: Use an energy method).

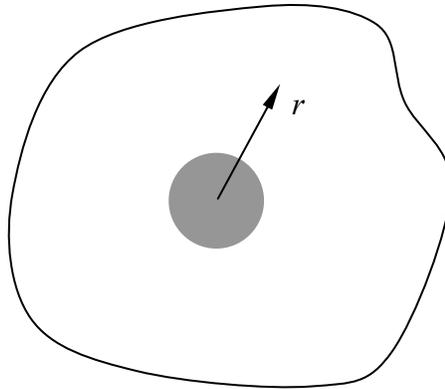


5. The displacement field outside of a sphere embedded in a large elastic body is

$$u_i = \left(\frac{a}{r}\right)^3 \varepsilon_{ijk} \omega_j x_k \quad \text{for } r = \sqrt{x_i x_i} > a$$

Here  $a$  is the sphere radius,  $\omega_i$  are the components of a constant vector,  $\varepsilon_{ijk}$  is the alternating symbol, and the position-vector  $x_i$  originates at the sphere center. The sphere is perfectly bonded to the surrounding material.

- Assume that the displacement field inside the sphere is linear and determine  $u_i(\mathbf{x})$  for  $r < a$ .
- Determine the strain field inside the sphere.
- Determine the tractions exerted on the sphere by the surrounding material.
- Formulate a boundary-value problem describing the situation.



## FIELD EQNS IN CIRCULAR CYLINDRICAL COORDINATES

### Equilibrium Equations

$$\begin{aligned}\frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_{r\theta}}{r \partial \theta} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + R &= 0 \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_\theta}{r \partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + \Theta &= 0 \\ \frac{\partial \sigma_{zr}}{\partial r} + \frac{\partial \sigma_{\theta z}}{r \partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\sigma_{zr}}{r} + Z &= 0\end{aligned}$$

### Strain-Displacement Relations

$$\begin{aligned}\varepsilon_r &= \frac{\partial u_r}{\partial r}, \quad \varepsilon_\theta = \frac{u_r}{r} + \frac{\partial u_\theta}{r \partial \theta}, \quad \varepsilon_z = \frac{\partial u_z}{\partial z} \\ \varepsilon_{r\theta} &= \frac{1}{2} \left( \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right), \quad \varepsilon_{\theta z} = \frac{1}{2} \left( \frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right), \quad \varepsilon_{zr} = \frac{1}{2} \left( \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right)\end{aligned}$$

### Stress-Strain Relations

$$\begin{aligned}\varepsilon_r &= \frac{1}{E} [\sigma_r - \nu(\sigma_\theta + \sigma_z)] + \alpha \Delta T \\ \varepsilon_\theta &= \frac{1}{E} [\sigma_\theta - \nu(\sigma_z + \sigma_r)] + \alpha \Delta T \\ \varepsilon_z &= \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_\theta)] + \alpha \Delta T \\ \varepsilon_{r\theta} &= \frac{(1+\nu)}{E} \sigma_{r\theta} \\ \varepsilon_{z\theta} &= \frac{(1+\nu)}{E} \sigma_{z\theta} \\ \varepsilon_{zr} &= \frac{(1+\nu)}{E} \sigma_{zr}\end{aligned}$$