ORBITS WRITTEN Q.E. (June 2012)

Each of the five problems is valued at 20 points. (Total for exam: 100 points)

PROBLEM 1

A) Summarize the content of the three Kepler's Laws.

B) Derive any two of the Kepler Laws.

PROBLEM 2

Two point masses, m_1 and m_2 , are in circular orbits about each other. The distance between them is a .

A) Derive the equations of motion of a particle in this gravitational field. The particle does not influence the motion of m_1 and m_2. Choose and describe any convenient reference frame for the equations of motion but state what you have chosen.
B) Derive any integrals of motion for the particle, and express them in terms of the physical constants given (and/or other constants derived from them) and the dynamical quantities referenced to the chosen reference frame.
C) Do equilibrium points for the particle exist? If so, provide a qualitative description, with a sketch to illustrate the location(s), and qualitatively state stability characteristics of the equilibrium points.

PROBLEM 3

Consider a Cassini-like spacecraft orbiting Saturn. The spacecraft is near the Titan sphere of influence boundary and is about to perform a flyby of Titan. The spacecraft Titan-centric velocity is the same as the spacecraft Saturn-centric velocity, which has been determined to be 8 km/sec. The spacecraft has no propellant, so it is unable to use the propulsion system to modify the orbit.

Assume the following for this problem:

- All celestial bodies can be gravitationally modeled as point masses.
- All planets and moons are in circular coplanar orbits
- Zero radius sphere of influence patched conic models are valid
- The Titan atmosphere extends to an altitude of 1000 km above the surface, and the spacecraft must stay outside the atmosphere
- The following parameters apply (where GM is gravitational parameter, a is the semi-major axis of the respective orbit, and R is body radius:

$$GM_{\text{Saturn}} = 4\times10^{7} \;\; \text{km}^3/\text{s}^2$$

$$GM_{\text{Sun}} = 1.3\times10^{11} \;\; \text{km}^3/\text{s}^2$$

$$GM_{\text{Titan}} = 9000 \;\; \text{km}^3/\text{s}^2$$

$$R_{\text{Titan}} = 2000 \;\; \text{km}$$

$$a_{\text{Titan}} = 1.2\times10^{6} \;\; \text{km}$$

$$a_{\text{Saturn}} = 1.5\times10^{9} \;\; \text{km}$$

$$a_{\text{Jupiter}} = 8\times10^{8} \;\; \text{km}$$

A) Is it possible for the spacecraft to escape Saturn after the flyby?

B) Is it possible to dispose of the spacecraft with an impact at Jupiter?

Note: For both parts A) and B):

- Provide a concise outline (bulleted or numbered) using words and relevant diagrams/equations that describes how to solve the problem
- Then solve the problem numerically

PROBLEM 4

A numerical integration for 10 days of the equations of motion of a GLONASS-like Earth orbit orbit produced the following mean values using a gravitational potential with terms in spherical harmonics through degree and order 3:

$$a = 25498 \text{ km}$$

$$e = 0.00145$$

$$i = 63^\circ$$

$$\dot{\Omega} = -0.0354^\circ \text{ / day}$$

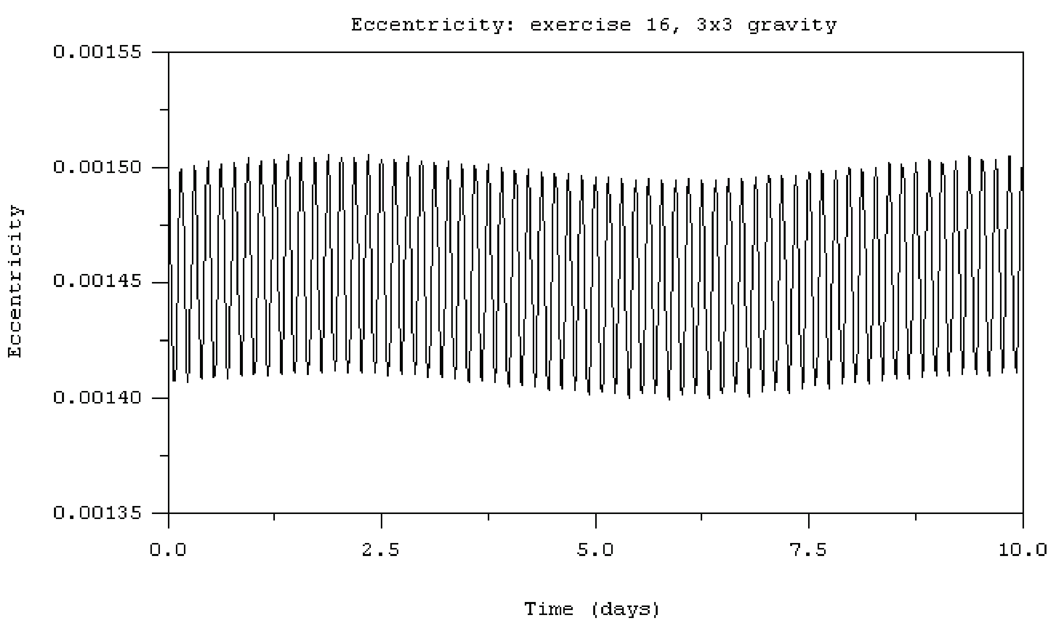
$$\dot{\omega} = 0.00119^\circ \text{ / day}$$

$$GM_{\text{Earth}} = 398600.4415 \text{ km}^3/\text{sec}^2$$

The latter two rates are mostly characterized as a linear variation. The evolution of eccentricity is shown below in the figure. The eccentricity plot exhibits two main periods, one of which is approximately 8 days. Also the semimajor axis exhibits a twice per orbit revolution variation with amplitude of about 2 km. The orbit period is ~40500 sec and the mean motion is $\sim 767.6^\circ / day$

What is (are) the primary cause(s) of the following specific variations:

A) The linear variation in Ω and ω?
B) The twice per orbit revolution variation in semimajor axis (plot not shown)?
C) The approximately 8 day period shown in e? (provide a justification)



PROBLEM 5

A celestial body has a constant, uniform mass density and the body can be represented by an ellipse of revolution. |

A) Write the gravitational potential function for this body using spherical harmonics with total mass M (you do not need to derive the expression but you may state an expression and define the terms you have used).

B) A satellite with mass m ($m << M$) orbits this celestial body. Assume the celestial body rotates about an axis z that coincides with the principal axis of maximum moment of inertia of the celestial body. Describe (and justify your description) the satellite's angular momentum per unit mass (is this vector constant?) in terms of (x,y,z) body-fixed components (define the directions of x and y). Note that the body rotates with a constant angular velocity vector that is coincident with z.

C) Let m represent a particle at rest on the surface of the ellipsoidal body, so the "weight" of the particle is $m\mathbf{g}$, where $\mathbf{g}$ is the "acceleration of gravity, $\mathbf{g}$ (vector)". Give the expression for $\mathbf{g}$ consistent with the preceding assumptions and determine the direction of $\mathbf{g}$.