Student number ______

PhD Written Qualifying Exam 2011

Solids

Department of Aerospace Engineering and Engineering Mechanics

PhD Written Qualifying Examination

Friday, June 10, 2011

9:00 am – 12 noon

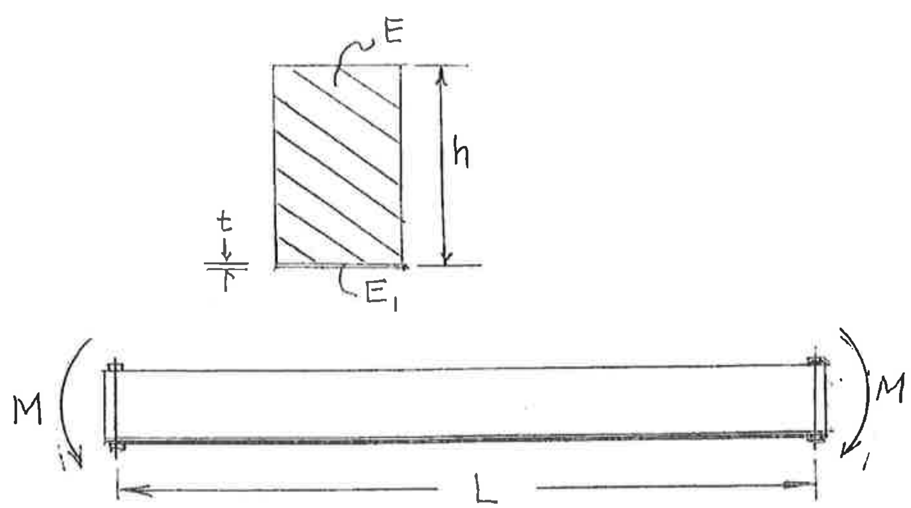
Answer all questions

THE UNIVERSITY OF TEXAS AT AUSTIN

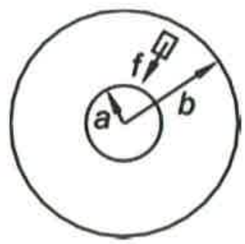
1. A rectangular cross section built-up beam consists of a more compliant upper section of depth h and elastic modulus E and a stiffer thin plate at the bottom of thickness $t << h$ and modulus E_1. The beam is bent in a way that places the stiffer plate in compression. You are required to find the moment-curvature relationship of this beam following the steps below. You can assume that the composite beam components deform in a compatible manner.

(*Hint:* Neglect the thickness of the thin plate in these calculations, i.e., assume the depth of the composite beam to be h).

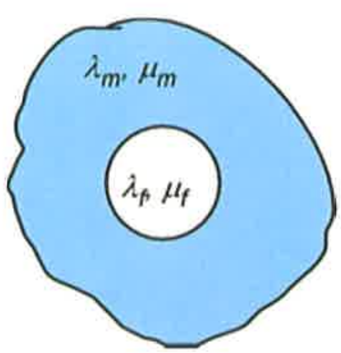
a) Assume that the thin plate carries a uniform stress and determine the location of the neutral axis of the beam.
b) Sketch the stress distribution in the beam if $E_1 = 10E$ and $h = 20t$.
c) Evaluate the moment-curvature relationship of the composite beam and determine the maximum stresses in the two materials for an applied moment M.
d) If now the plate is not bonded to the upper section, but forced to conform to the same strain by connectors at the ends of the beam (see figure), what is the contact stress that develops between the upper section and the plate strip?



2. A long, thick-walled cylinder is only subjected to the radial body force $f = \frac{g}{r}$ shown below.
   a) Explain why the stress function $\phi = A\ln r + Br^2 \ln r + Cr^2 + D$, which was developed for zero body forces, is appropriate in this case.
   b) Determine the stresses in the cylinder. Note that when the rotation is zero, $(\sigma_r + \sigma_\theta)$ is constant.
   c) Determine the displacements at the outer radius of the cylinder to within a rigid body motion.

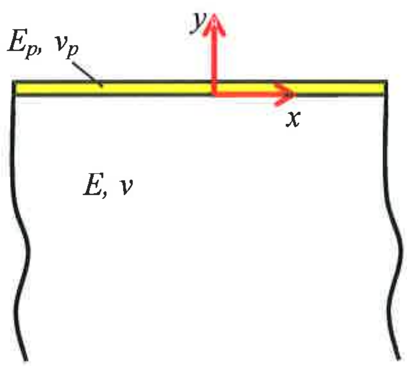


3. A circular cylindrical fiber with elastic Lame constants λ_f and μ_f is embedded in a matrix with elastic Lame constants λ_m and μ_m. Under certain conditions, the fiber is uniformly strained so that $\varepsilon_{rr} = \varepsilon_{\theta\theta} = \varepsilon_{rz} = \alpha$, $\varepsilon_{zz} = -2\alpha$, and $\varepsilon_{r\theta} = \varepsilon_{\theta z} = 0$, with a prescribed α; the subscripts refer to cylindrical coordinates naturally aligned with the fiber.
   a) Assuming that the fiber and matrix are perfectly bonded, formulate the continuity conditions.
   b) Determine the stress state in the matrix at points along the circular fiber-matrix interface.
   c) Perform the calculations by assuming that $\lambda_f = \mu_f$ and $\lambda_m = \mu_m$.
   d) Verify the solution by considering a particular case for which the solution can be easily constructed.

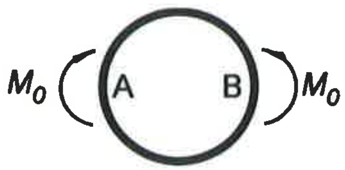


4. A thin elastic plate of thickness h coats the surface of an elastic half plane (see figure). The plate is subject to a compressive residual stress (σ) in the x-direction. At a critical stress, the plate becomes unstable and buckles. To find the critical stress, you may assume that the plate deforms sinusoidally, $u_y = A\cos(kx)$, and that the interface is perfectly bonded. The problem is solved in two steps. First, an elastic half-plane with a periodic surface displacement is considered. Solving the plane-strain elasticity problem yields the normal traction on the surface to be $t_y = \frac{kE}{2(1-\nu^2)} u_y$, where E and ν are Young's modulus and Poisson's ratio of the half-plane. In the second step, for which you are responsible, the critical stress is determined by coupling the linear plate equation, i.e., $t_y = -\frac{E_p h^3}{12(1-\nu_p^2)} \frac{d^4 u_y}{dx^4} - \sigma h \frac{d^2 u_y}{dx^2}$, with the half-plane solution, where E_p and ν_p are the Young's modulus and Poisson's ratio of the plate.

(a) State any additional assumptions that need to be made for this analysis.
(b) Determine the critical stress as a function of kh.



5. A slender uniform ring (radius R, bending rigidity EI) is loaded by moments M_0 at diametrically opposite points A and B as shown in the figure. Determine the amount of rotation at A and/or B.



Equations

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\sigma_{r\theta}}{r\partial\theta}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}+f_r=0$$

$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{\partial\sigma_\theta}{r\partial\theta}+\frac{\partial\sigma_{\theta z}}{\partial z}+\frac{2\sigma_{r\theta}}{r}+f_\theta=0$$

$$\frac{\partial\sigma_{zr}}{\partial r}+\frac{\partial\sigma_{\theta z}}{r\partial\theta}+\frac{\partial\sigma_z}{\partial z}+\frac{\sigma_{zr}}{r}+f_z=0$$

$$\nabla^2\left(\sigma_r+\sigma_\theta\right)=-(1+\nu)\left(\frac{f_r}{r}+\frac{\partial f_r}{\partial r}+\frac{1}{r}\frac{\partial f_\theta}{\partial\theta}\right)$$

$$\sigma_r=\left(\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right);\sigma_\theta=\left(\frac{\partial^2\phi}{\partial r^2}+rf_r\right);\sigma_{r\theta}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)$$

$$\varepsilon_r=\frac{\partial u_r}{\partial r},\ \varepsilon_\theta=\frac{u_r}{r}+\frac{\partial u_\theta}{r\partial\theta},\ \varepsilon_z=\frac{\partial u_z}{\partial z}$$

$$\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right),\ \varepsilon_{\theta z}=\frac{1}{2}\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\theta}\right),\ \varepsilon_{zr}=\frac{1}{2}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)$$

$$\omega_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r}-\frac{1}{r}\frac{\partial u}{\partial\theta}$$

$$\varepsilon_{ij}=\frac{1+\nu}{E}\sigma_{ij}-\frac{\nu}{E}\sigma_{kk}\delta_{ij};\qquad \sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\varepsilon_{kk}\delta_{ij}$$