Student number ______

PhD Written Qualifying Exam 2011

Fluids

Fluid Mechanics PhD Qualifying Examination
June 2011

READ THESE INSTRUCTIONS.

Answer all five questions. Please write on one side of the paper only and put your Code Number and appropriate question number on every sheet. **Begin each question on a separate sheet of paper**. To obtain complete credit, your work must be neat and your complete procedure shown. Draw neat sketches and list your assumptions. Ask for clarification if the meaning of a question is unclear to you.

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1. Consider a steady laminar zero-pressure gradient boundary layer developing over a plate with the fluid freestream velocity being U. The Blasius equation describing this flow is given by

$$f''' + \frac{1}{2}ff'' = 0,$$

where $f = f(\eta)$ and $f' = u(y)/U$. Here, $\eta = \frac{y}{\delta(x)}$, where $\delta(x)$ is the boundary layer thickness as a function of location streamwise location (x).

   (a) Starting from 2D boundary layer equations and using scaling arguments, express the dependance of boundary layer thickness on the streamwise distance.

   (b) Derive the streamfunction expressed in terms of f.

   (c) Provide the boundary conditions needed to solve the Blasius equation.

   (d) Explain a numerical method for solving the Blasius equation.

2. Consider the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

where α is the diffusivity. Consider an initial field of u that has a many different wavenumbers associated with it.

$$u(x,0) = \sum_{k=1}^{N} \hat{u}^k e^{ikx}$$

   (a) How will the wavenumbers $\hat{u}^k$ evolve with time in the solution to the heat equation?

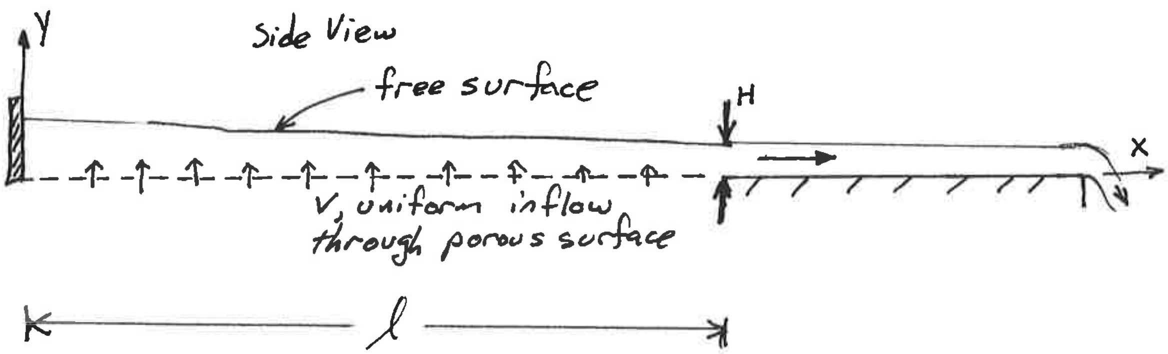
   (b) If a second-order accurate finite difference approximation is used to approximate the spatial derivative, how will the wanumbers evolve?

   (c) Any grid can support only a finite number of wavenumbers. Show what happens to the evolution of the wavenumbers that are close to the maximum resolvable values on a grid. Use this result to explain potential implications for solving turbulent flow using finite difference approximations.

3. A viscous liquid of density ρ and viscosity μ is allowed to slowly rise up through the porous floor of a rectangular channel sitting on a table top. The inflow vertical velocity, v, is uniform over the entire porous area of width w and length l. The fluid layer thickness h(x) decreases with increasing distance x from the left hand channel boundary. The channel extends to the right beyond the end of the porous region (at $x=l$). At location $x=l$ fluid layer thickness $h(l) = H$.

In terms of the known parameters v, w, l, H, ρ and μ and the unknown layer thickness $h(x)$, what are

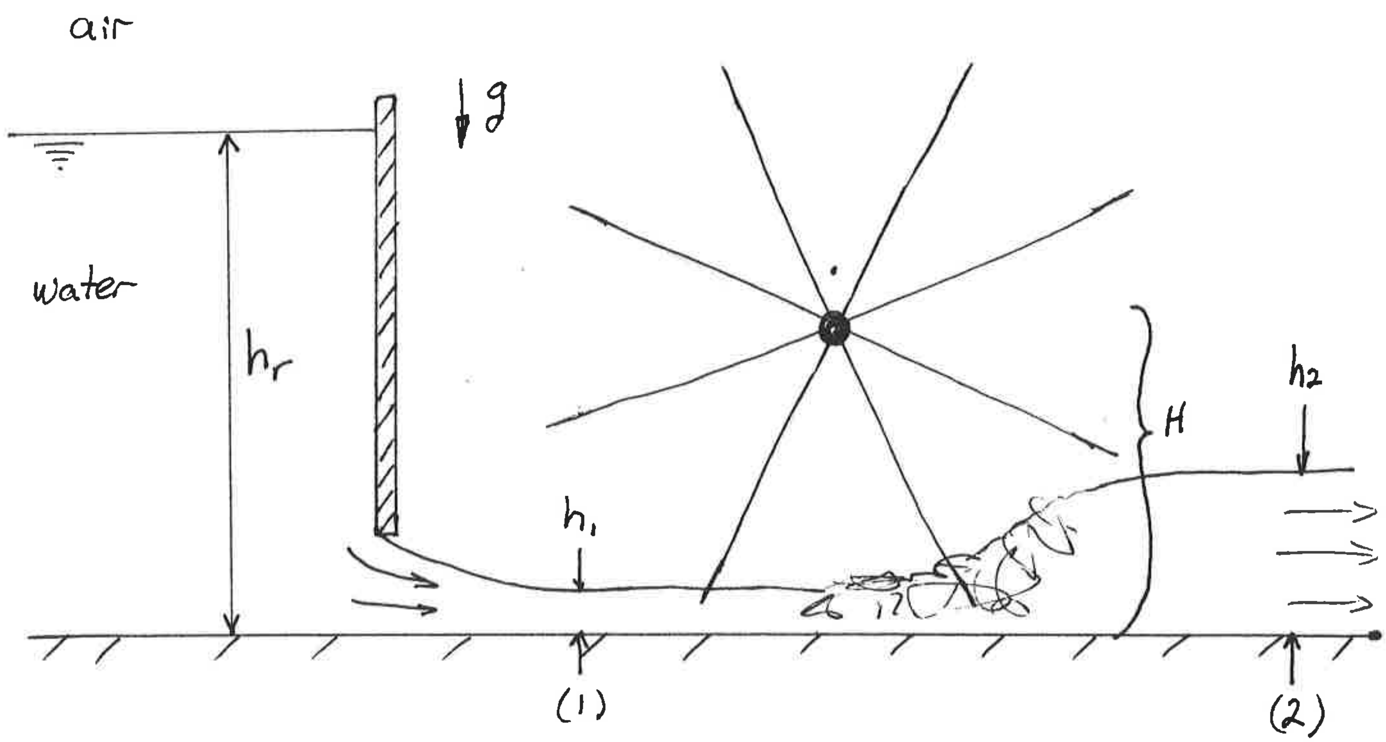
a) The volume flow rate $Q(x)$ to the right?
b) The pressure distribution in the layer $p(x,y)$?
c) The horizontal velocity distribution $u(x,y)$ at any distance x from the left end (assume steady laminar flow)?
d) Then, from these expressions in (a),(b), and (c) and the known parameters, derive an explicit expression for $h(x)$.
e) Finally, in Barton Springs pool water flows up out of the ground from subterranean aquifers at about $10 m^3/s$ and flows away. The pool is several tens of meters in width and length. Comment on how applicable the above tabletop model and mathematical analysis would be to the actual conditions at Barton Creek.



4. The paddle wheel of an old grist mill is located beside a sluice gate in a reservoir. The gate, stream and paddle wheel all have a width (into the page) of 1m. The water surface in the reservoir is a measured height $h_r = 5$m above the solid surface of the stream bed. The water accelerates under the sluice gate (the solid vertical barrier shown in the figure) and approaches the paddle at a depth of $h_1 = 1$m and leaves the paddle downstream at a greater depth of $h_2 = 2$m. The central axis of the wheel is located a distance H=4m above the stream bed. The flow from the reservoir to station 1 is smooth and effectively inviscid. But across the region of the paddle wheel there is much turbulence and viscous dissipation. However, even so, the viscous shear force on the smooth stream bed is negligibly small.

Calculate:

a) The water flow velocities V_1 and V_2 at stations 1 and 2.
b) The horizontal force F_x exerted by the stream on the paddle wheel.
c) The torque exerted by the flow upon the wheel.



5. A cylindrical turret mounted to the fuselage of a rocket is being used to provide live video feedback of the spacecraft's performance during launch. The radius (R) and length (l) of the turret is small relative to the diameter of the rocket fuselage, and so it can be approximated as a half cylinder on a flat plate. If the incoming flow (U_∞) is frictionless, steady, incompressible and uniform across the length of the cylinder, determine the following:

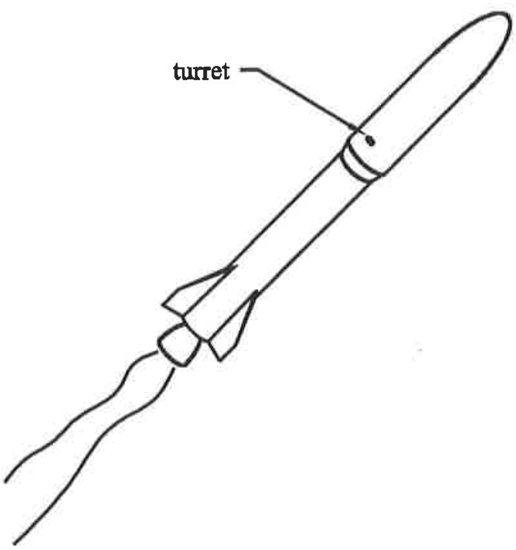
   a.) Using potential flow, derive an expression for the velocity field over the turret and show (mathematically) the location of the stagnation points.

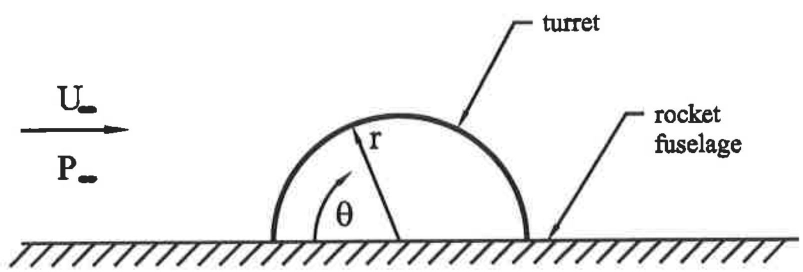
   b.) Obtain an expression for the pressure coefficient along a streamline forming the turret surface and neatly draw a sketch of this profile.

   c.) Identify points on the turret where the surface pressure (P) equals the freestream pressure (P_∞).

   d.) Determine the net lifting force tending to pull the turret away from the fuselage. Express your final answer in terms of a suitable lift coefficient.

   d.) Sketch the streamlines of this flow that would be expected using potential flow theory. In a separate drawing, sketch the streamlines that one would expect to actually occur (the real world) and point out the key differences with respect to the first sketch.





Note, $\phi_{doublet} = \frac{K}{2\pi r}\cos(\theta)$

ADDENDUM

The continuity and Navier-Stokes Equations in cylindrical coordinates (r, θ, z) for incompressible flow with constant properties are

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right)$$

$$= F_r - \frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right)$$

$$= F_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right)$$

$$= F_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$