Fluid Mechanics PhD Qualifying Examination.
June 2012

READ THESE INSTRUCTIONS:

Answer all **five** questions. Please write on one side of the paper only and put your Code Number and appropriate question number on every sheet. **Begin each question on a separate sheet of paper.** To obtain complete credit, your work must be neat and your complete procedure shown. Draw neat sketches and list your assumptions. Ask for clarification if the meaning of a question is unclear to you.

1. A flame arrestor is used in pipelines and engines to prevent the upstream propagation of a flame should there be fuel fumes present in the incoming air. (The strain rate and heat absorption are too high to permit the propagation of the flame.) Consider a simple flame arrestor made of a series of very thin parallel plates aligned with the inlet flow (V) as seen in the figure below. The plate spacing is h and the plate length is L. The viscosity is μ and density is ρ.

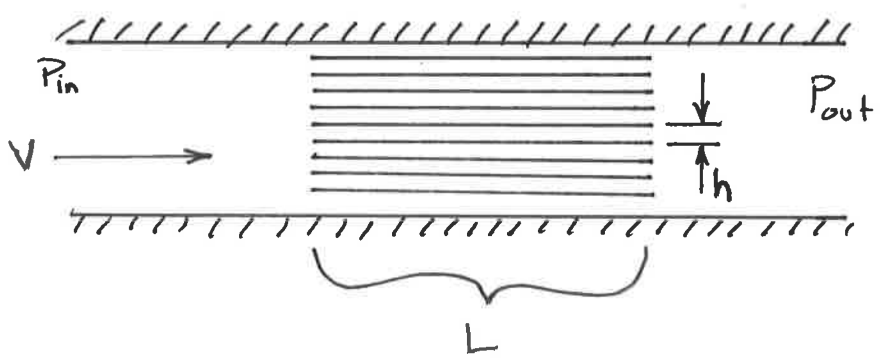
   Assuming the flow is incompressible, determine expressions for the pressure drop $P_{in} - P_{out}$ between the incoming stream and the outgoing stream for the limiting cases of:

   a.) low flow velocity in which case the flow between each pair of plates is well represented as a fully developed plane Poiseuille flow.

   b.) high velocity flow where, instead, a thin Blasius boundary layer develops on the top and bottom of each plate surface as though it were not influenced by the adjacent plates. It might help to recall that the Blasius shear stress is

$$\tau_w = 0.3321\sqrt{\frac{\rho\mu V_\infty^3}{x}}.$$

   c.) Finally, determine the Reynolds number Vh/ν at which the pressure drops found in (a.) and (b.) are equal in the case where $L = 10h$.



2. Consider incompressible, zero pressure–gradient laminar flow over a flat plate. The fluid viscosity is μ and density is ρ. Suppose the velocity profile is approximated by:

$$\frac{u}{U_e} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2.$$

Determine the following:

a.) The boundary layer thickness $\delta(x)$.

b.) The skin friction coefficient $C_f(x)$.

c.) What is the boundary layer shape factor, $H = \delta^*/\theta$? The shape factor for a Blasius boundary layer is 2.59. What does the value you obtain for H imply about the stability of this flow compared to the Blasius case?

d.) Find an expression for the drag, D, on one side of a plate of length L and span b.

3. A vortex and sink can be combined to simulate a tornado where Γ and Λ describe the circulation and sink strengths of the tornado, respectively.

   a.) Determine an expression for the flow velocity at any arbitrary point in the tornado.

   b.) Write down an expression for the pressure difference (relative to ambient) along any streamline for arbitrary circulation and sink strengths.

   c.) A crude relationship between the dry bulb temperature (T; total temperature) and the dew point temperature (T_d) for water vapor is expressed as:

$$T_d = T - \frac{100\% - RH}{5}.$$

   where RH is the relative humidity in the air and the dew point temperature is equal to the static temperature. If the relative humidity of the ambient air being entrained by the vortex is 75%, the dry bulb temperature is 30°C, and the Vortex and Sink strengths are 4,000m^2/s and 200m^2/s, respectively, determine the distance from the center of the tornado where water vapor is expected to condense.

   d.) Is the flow incompressible at this location?

   e.) Suppose the average droplet size is expected to be $10\mu m$ in diameter. Estimate the minimum distance (radius) from the tornado axis that these particles will reach.

   f.) What would be the particle Reynolds number at this location?

4. For the following questions, $\vec{U}$ refers to the velocity vector, while $\vec{\omega}$ refers to the vorticity vector.

   a.) Consider an incremental displacement along a streamline given by $\vec{dx}$. Show that

   $$\left(\vec{U} \times \vec{\omega}\right) \cdot \vec{dx} = 0.$$

   b.) Prove

   $$\vec{U} \cdot \nabla \vec{U} = \nabla(\vec{U} \cdot \vec{U})/2 - \vec{U} \times \vec{\omega}.$$

   c.) Using the above two relations, derive Bernoulli's equation starting from Navier-Stokes equations for steady incompressible flows with negligible viscous forces.

5. Consider the one-dimensional scalar convection equation in a periodic domain given by

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

where c is a constant scalar speed. Consider an initial field of u that has many different wavenumbers associated with it:

$$u(x,0) = \sum_{k=1}^{N} \hat{u}_k e^{ikx}$$

a.) How will the wavenumbers $\hat{u}_k$ evolve with time in the solution to the convection equation?

b.) If a second-order accurate finite difference approximation is used to approximate the spatial derivative, how will the wavenumbers evolve? Compare to the exact solution and discuss the impact of finite difference approximation.

ADDENDUM

The continuity and Navier-Stokes Equations in cylindrical coordinates (r, θ, z) for incompressible flow with constant properties are

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right)$$

$$= F_r - \frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right)$$

$$= F_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right)$$

$$= F_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right)$$