Summer 2004 PhD Qualifying Examination
Control Theory

(100 points, 180 minutes)
Closed Book and Closed Notes

1. (20 points) Consider the linear time-invariant system shown in Figure 1.

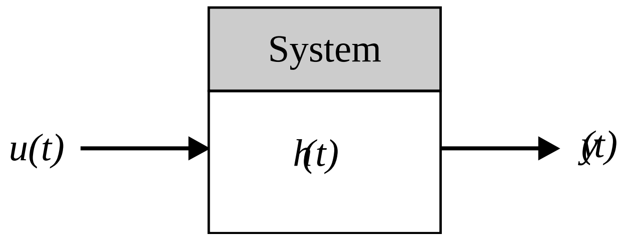

Figure 1: Linear time-invariant system for Problem 1.

The system impulse response is

$$h(t) = \begin{cases} ke^{\sigma t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where k and σ are given real constants. The system is initially at rest. Answer the following questions and justify your answer in each case:

(a) (5 points) Is the system causal or noncausal?

(b) (5 points) For what values of k and σ is the system bounded-input, bounded-output stable?

(c) (5 points) For a given arbitrary input, $u(t)$, what is the response, $y(t)$ of the system?

(d) (5 points) Suppose that $u(t)$ is a unit step input for $t \geq 0$. Compute the output $y(t)$ for $t \geq 0$.

2. (20 points) Consider the system model depicted in Figure 2.

(a) (5 points) Determine the characteristic equation of the closed-loop system.

(b) (5 points) Compute the sensitivity of the closed-loop transfer function to variations in the plant transfer function $G(s)$. Explain the physical significance of the sensitivity and describe how you would design the controller $G_c(s)$ to reduce the sensitivity?

(c) (10 points) If possible, compute K_I and K_P so that the following design specifications are satisfied:

1. The closed-loop system is stable.
2. The output $y(t)$ tracks a ramp input $r(t)$ with a zero steady-state error;
3. The percent overshoot to a unit step is less than or equal to 5%.

If you determine that it is not possible to meet the design specifications, explain your reasoning carefully.

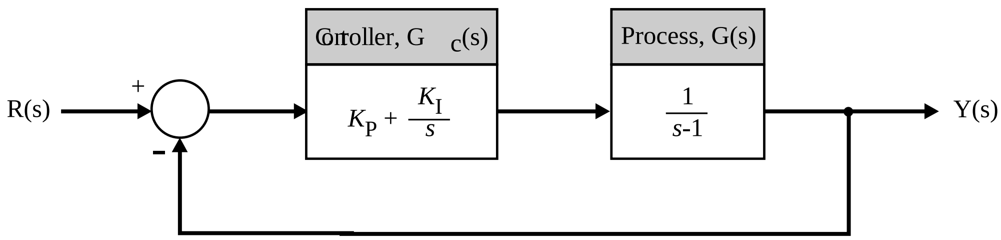

Figure 2: Block Diagram for Problem 2.

3. (10 points) Consider the system shown in Figure 3.

   (a) (5 points) Suppose that $K = 1$. Using Bode frequency response methods, estimate the phase margin of the system. Is the closed-loop system stable?

   (b) (5 points) Using root locus methods, determine the acceptable values of K that result in closed-loop stability.

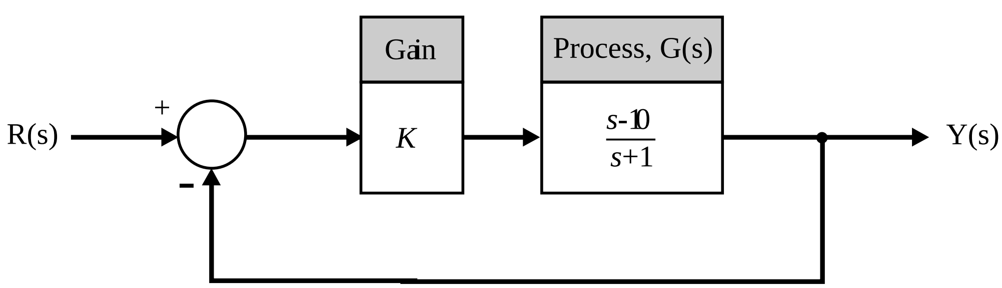

Figure 3: Block Diagram for Problem 3.

4. (10 points) A continuous-time LTI system has a state space model given by the matrices A, B, C and D as follows

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ \gamma \end{bmatrix}, \quad C = [c_1\ c_2], \quad D = 0.$$

Note that γ, c_1 and c_2 are arbitrary real number constants.

(a) (5 points) Evaluate the system transfer function.

(b) (5 points) Discuss the pole and zero locations as $\gamma \to 0$. Interpret this in terms of controllability.

5. (15 points) Consider a generic SISO LTI system state space description (A, B, C, D), where $B = 0$ and $D = 0$.

   (a) (5 points) Determine a pair of matrices A and C and an initial state $x(0)$ such that $y(t) = 5, \ \forall t \geq 0$. Note that the dimension of the state vector is left unspecified in this problem.

   (b) (5 points) Repeat part (a) above for $y(t) = -3 + 2\sin(0.5t + \pi/3)$.

   (c) (5 points) Try to generalize the problem.

6. (25 points) A state space model for a linear system is given by (A,B,C,0), where

$$A = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad C = [-1 \ \ \alpha], \quad \alpha \in \mathcal{R}.$$

   (a) (5 points) Determine, if it exists, a full state-feedback gain K such that the closed-loop poles are located at -5 and -6.

   (b) (5 points Determine, if it exists, a full state-feedback gain K such that the modes of the closed-loop response have the generic form $\beta_1 e^{-2t}\cos(0.5t + \beta_2)$ in terms of arbitrary constants β_1 and β_2.

   (c) (5 points) Find the range of values of α for which the system is completely observable.

   (d) (10 points) Choose a particular value of α that makes the system observable, and build an observer, whose error decays faster than e^{-5t}.