

A note on significant digits and precision

Here is a FAQ and my response:

- > *I have a question of when to truncate an answer to the number of*
- >*significant digits. A problem may have 3 steps and if I truncate at each*
- >*step the answer is different than if I didn't truncate. Initially I thought*
- >*that one should not truncate until the end, but what is the end of a problem*
- >*for one engineer is oftentimes the beginning of a problem for another. What*
- >*advice can you offer?*
- >

This is a tough question to answer in general. As you note, there can be a problem with "accumulated round-off error". Of course it is not always clear that this is an "error". As a rough rule of thumb, I suggest you carry four (in general $n+1$) digits in intermediate calculations and then round-off the final answer to 3 (or n) digits. The only time you may have to carry more digits is when you are taking the difference of two large numbers, as there is a rapid loss of precision when the two numbers are almost equal. This should be a red-flag that one should reformulate the computation procedure if possible to calculate the difference directly or carry more digits than one would normally.

A typical example of loss of precision is calculating the pressure difference between two stations in a flow (in a wind-tunnel for example). If the two pressure p_1 and p_2 are gauge pressures, say $p_1 = 102$ Pa (gauge) and $p_2 = 96$ Pa (gauge), then $p_1 - p_2 = 6(\pm 1?)$ Pa. Now consider what happens if you decide to convert everything to absolute pressure (P), and you assume that $P_{\text{atm}} = 101$ kPa. Then you must carry $P_1 = 101102$ Pa (??) and $P_2 = 101096$ Pa (??) in your calculations to maintain the same (low) level of precision of the previous calculation. If you try to round off to 3 or 4 digits in the intermediate calculation, arguing that P_{atm} is only known to 3 digits, then you lose all information on the pressure difference. The key here is to note that the gauge pressure is known to 3 digits at the Pascal level, while the absolute pressure is known to only 3 digits too but at the kPa level. Now suppose you wanted to compute the mass flow rate of air through the wind tunnel, and you measure the temperature of the air as 298 K at station 1, i.e. $T_1 = 298$ K. Then density $\rho_1 = p_1/(R T_1) = 1.18$ kg/m³ and this is the same as ρ_2 assuming constant T because the absolute pressure variation (which determines density variations) is insignificant. Hence the flow is incompressible to the 1% level or better. However, when computing pressure differences one should not add in the atmospheric pressure numerically because all it will do is degrade precision, and the gauge pressure difference is needed to determine the flow velocity.

If the result is really important (lives depend on it, or lots of money) then a clear statement of the precision of the final answer ($x.xx \pm y$) is important. Estimating y may take some effort. A careful analysis of all sources of error and uncertainty in data, model, assumptions, and computational procedure is required.